



# Review IB Math AI SL

Topic 1: Number and Algebra  
by @TUTORIO.MATH

# Approximation and Estimation

Today, you'll be able to:



Rounding (d.p. and s.f.)



Measurement bounds



Error: Absolute and Percentage



Scientific Notation





# Rounding (d.p. and s.f.)

$(n+1)^{\text{th}}$  digit

0, 1, 2, 3, 4 **round down** (same digit)

5, 6, 7, 8, 9 **round up** (digit + 1)

\***Significant figures:** start from first (most left) non-zero digit!

|            | 2 d.p.                               | 3 s.f.  |
|------------|--------------------------------------|---|
| e.g. 3.413 | $3.\underline{41}3 \approx 3.41$     | $\underline{3}.\underline{41}3 \approx 3.41$    |
| -47.569    | $-47.\underline{56}9 \approx -47.57$ | $-\underline{47}.\underline{56}9 \approx -47.6$ |
| 0.03527    | $0.\underline{03}527 \approx 0.04$   | $0.\underline{035}27 \approx 0.0353$            |

Estimate using a one figure approximation:

a.  $384 \times 28$

b.  $6.92 \times 0.792$

c.  $37954 : 211$

$$\begin{aligned} \text{a) } & \underline{3}84 \times \underline{2}8 \\ & = 400 \times 30 \\ & = 12\,000 \end{aligned}$$

$$\begin{aligned} \text{b) } & \underline{6}.\underline{9}2 \times 0.\underline{7}92 \\ & = 7 \times 0.8 \\ & = 5.6 \end{aligned}$$

$$\begin{aligned} \text{c) } & \underline{3}7954 : \underline{2}11 \\ & = 40000 : 200 \\ & = 200 \end{aligned}$$



# Practice 1

$$r = \frac{d}{2}$$

A circle has a diameter 7.37 cm. Find the area of the circle, rounding your answer to:

- 3 significant figures
- 4 decimal places

$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= \pi \left( \frac{7.37}{2} \right)^2 \\ &\approx 42.6603935 \end{aligned}$$

$$\text{a) } A \approx \underline{42.660}3935$$

$$\therefore A \approx 42.7 \text{ (3 s.f.)}$$

$$\text{b) } A \approx 42.\underline{66039}35$$

$$\therefore A \approx 42.6604 \text{ (4 d.p.)}$$



## Practice 2

## Measurement BOUNDS

$$x_{min} \leq x < x_{max}$$

$$x_{min} = x - \frac{p}{2}$$

$$x_{max} = x + \frac{p}{2}$$

$p$  is precision

## Absolute and % ERRORS

$$\text{Percentage} : \frac{|v_a - v_e|}{v_e} \times 100\%$$

$$\text{Absolute} : |v_a - v_e|$$

$v_a$  = approximate value

$v_e$  = exact value



# Practice 1



Ben measures a box as 15 cm x 12 cm x 8 cm, rounded to the nearest centimetre.

a. Use Ben's measurements to estimate the box volume

$$V \approx 15 \cdot 12 \cdot 8$$
$$\therefore V \approx 1440 \text{ cm}^3$$

$\hookrightarrow V_a$

The actual dimensions of the box are 15.2 cm x 11.9 cm x 8.2 cm

b. Find the actual volume of the box

$$V = 15.2 \cdot 11.9 \cdot 8.2$$
$$\therefore V = 1483.216 \text{ cm}^3$$

$\hookrightarrow V_e$

c. Find the absolute and percentage error in Terry's estimate

$$\begin{aligned} \text{Absolute error} &= |V_a - V_e| \\ &= |1440 - 1483.216| \\ \therefore \text{Absolute error} &\approx 43.216 \text{ cm}^3 \end{aligned}$$
$$\begin{aligned} \text{Percentage error} &= \frac{|V_a - V_e|}{V_e} \cdot 100\% \\ &= \frac{43.216}{1483.216} \cdot 100\% \\ \therefore \text{Percentage error} &\approx 2.9\% \text{ (1 d.p.)} \end{aligned}$$





# Practice 2

The radius of a circle is measured as 7 cm, rounded to the nearest centimetre.  $p = 1 \text{ cm}$   $\rightarrow V_a$

a. Use this measurement to estimate the area of the circle

$$V = \pi \cdot 7^2$$

$$\therefore V = 49\pi \text{ cm}^2 \approx 154 \text{ cm}^2 \text{ (nearest cm}^2\text{)}$$

b. Find the boundary values for the area of the circle

$$r_{\min} \leq r < r_{\max} \qquad A_{\min} \leq A < A_{\max}$$

$$7 - \frac{1}{2} \leq r < 7 + \frac{1}{2} \qquad \pi \cdot (6.5)^2 \leq A < \pi \cdot (7.5)^2$$

$$6.5 \leq r < 7.5 \qquad \therefore 42.25\pi \text{ cm}^2 \leq A < 56.25\pi \text{ cm}^2$$

$$\therefore 133 \text{ cm}^2 \leq A < 177 \text{ cm}^2$$

c. Find the maximum percentage error in the estimate

↳ Test percentage error on (i) min  $V_e$  & (ii) max  $V_e$ , then pick the **HIGHEST** one!

$$(i) \% \text{ error} = \frac{|49\pi - 42.25\pi|}{42.25\pi} \cdot 100\% \qquad (ii) \% \text{ error} = \frac{|49\pi - 56.25\pi|}{56.25\pi} \cdot 100\%$$

$$\approx 16.0\% \quad \& \qquad \approx 12.9\%$$

∴ Max. percentage error of the estimate area is 16.0%



# Scientific Notation

$$a \times 10^k$$

$$\{1 \leq a < 10, k \in \mathbb{Z}\}$$

Exponent is **positive**

$$2.3 \times 10^6 = 2300000.$$

1 2 3 4 5 6

Exponent is **negative**

$$3.78 \times 10^{-5} = 0.0000378$$

1 2 3 4 5



# Practice 1

Write in scientific notation:

a. 44000

b. 0.0000684

c. 526000000

$$a) \underline{4}.\overset{1}{4}000 = 4.4 \times 10^4$$

$$b) 0.\overset{5}{0000}\underline{6}84 = 6.84 \times 10^{-5}$$

$$c) \underline{5}.\overset{8}{26}000000 = 5.26 \times 10^8$$



## Practice 2

Use calculator to evaluate, then give your answers in scientific notation.

$$(3.57 \times 10^6) \times (2.38 \times 10^3) = 8.4966 \times 10^9$$

$$\frac{4.61 \times 10^{-7}}{3.45 \times 10^8} = 1.34 \times 10^{-15}$$



**See you on next  
video!**

For private tutoring inquiries, please reach me on  
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