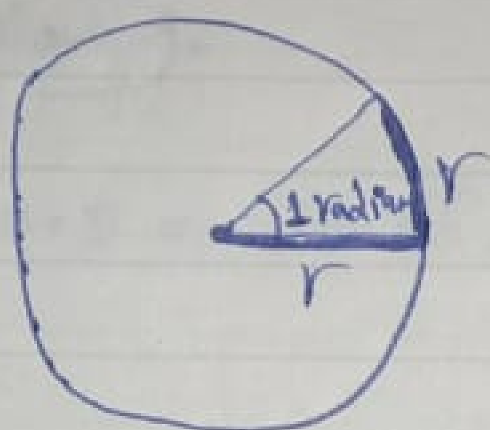
Trignometric Methods Assignment

3.1

* one radian is the angle of an arc created by wrapping the radius of a circle around its circumference.
The radius 'r' fits around the circumference of a circle exactly 2π times. That is why the circumference of a circle is given by: $C = 2\pi r$

محيطها



3.2

(a) $145^\circ = 145^\circ \times \frac{\pi}{180} = \frac{29}{36}\pi \text{ rad} = 2.53 \text{ rad}$

(b) $19^\circ = 19^\circ \times \frac{\pi}{180} = 0.33 \text{ rad}$ (c) $526^\circ = 526 \times \frac{\pi}{180} = 9.18 \text{ rad}$

(d) $0.85 \text{ rad} = 0.85 \times \frac{180}{\pi} = 48.7^\circ$

(e) $\pi/9 = \frac{\cancel{\pi}}{9} \times \frac{180}{\cancel{\pi}} = 20^\circ$ (F) $\frac{3\pi}{4} = \frac{3\cancel{\pi}}{4} \times \frac{180}{\cancel{\pi}} = 135^\circ$

3.3. Find the arc length and area for each of the following, showing working:

a) A sector of radius 5 m and angle 1 radian.

$\boxed{\text{Arc length } s = \theta r = 1 \times 5 = 5\,m}$

Arc area $= \frac{1}{2}\theta r^2 = \frac{1}{2} \times 1 \times (5)^2 \Rightarrow \boxed{\text{Area} = 12.5\, m^2}$

b) $r = 2.3\,m$, $\theta = \pi$ rad

$$\boxed{s = \theta r = \pi \times 2.3 = 7.22\ m}$$

$$Area = \frac{1}{2}\theta r^2 = \frac{1}{2} \times \pi \times (2.3)^2 \approx 8.31\,m^2$$

$$\boxed{Area = 8.31\ m^2}$$

4.1, 6.3 (i), Find all the values of θ between 0° and 360° that satisfy the following.

a) $\cos\theta = 0.5 \Rightarrow \theta = 60^\circ, 300^\circ$ (Intersections of 0.5 with $\cos\theta$ graph)

b) $\cos\theta = -0.7 \Rightarrow \theta \approx 135^\circ, 225^\circ$
(Intersections of -0.7 with $\cos\theta$ graph)

c) $\cos 3\theta = 0.7 \Rightarrow$ ~~$3\theta = 45, 405, 135, 225, 315, 345$~~

~~$\therefore \theta = 5^\circ, 35^\circ, 45^\circ, 75^\circ, 85^\circ, 115^\circ$~~

$$\boxed{\begin{array}{l} 3\theta = 45, 315, 405, 675, 765, 1035 \\ \theta = 15, 105, 135, 225, 255, 345 \end{array}}$$

d) $\cos(\theta - 30^\circ) = -0.6 \Rightarrow (\theta - 30) = 127, 233$

$\therefore \theta - 30 = 127 \rightarrow \boxed{\theta = 157^\circ}$

$\theta - 30 = 233 \rightarrow \boxed{\theta = 263^\circ}$

4.1, 6.3 (ii) using the graphs given, Find the values of θ between 0 and 2π radians

(a) $\sin\theta = 0.5 \Rightarrow \theta = 30^\circ, 150^\circ \Rightarrow 30^\circ \times \frac{\pi}{180} = \frac{\pi}{6}, \ 150^\circ \times \frac{\pi}{180} = \frac{5\pi}{6}$

$\boxed{\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}}$

(b) $\sin\theta = -0.7 \Rightarrow$ From graph $\theta = 225^\circ, 315^\circ$

$225^\circ \times \frac{\pi}{180} = \frac{5}{4}\pi$ rad, $315^\circ \times \frac{\pi}{180} = \frac{7}{4}\pi$ rad

$\boxed{\theta = \frac{5}{4}\pi, \frac{7}{4}\pi}$

(c) $\sin 3\theta = 0.7 \Rightarrow \boxed{3\theta = 45^\circ, 135^\circ, 405^\circ, 495^\circ, 765^\circ, 855^\circ}$

$\boxed{\therefore \theta = 15^\circ, 45^\circ, 135^\circ, 165^\circ, 255^\circ, 285^\circ}$

$15^\circ \times \frac{\pi}{180} = \frac{\pi}{12}$ rad, $45^\circ \times \frac{\pi}{180} = \frac{\pi}{4}$ rad, $135^\circ \times \frac{\pi}{180} = \frac{3\pi}{4}$

$\cancel{\frac{165}{180}}$ $165 \times \frac{\pi}{180} = \frac{11\pi}{12}$, $255^\circ \times \frac{\pi}{180} = \frac{17}{12}\pi$ rad, $285 \times \frac{\pi}{180} = \frac{19}{12}\pi$

$\boxed{\theta = \frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}}$

(d) $\sin(\theta + \pi/8) = -0.4$

$(\theta + \underset{22.5}{\cancel{\pi/8}}) = 202.5^\circ, 337.5^\circ$

$\sin(202.5) \simeq -0.4$
$\sin(337.5) = -0.4$
for check only

$\boxed{\therefore \theta = 180^\circ, 315^\circ}$

$180^\circ \times \frac{\pi}{180} = \pi$, $315^\circ \times \frac{\pi}{180} = \frac{7}{4}\pi$

$\boxed{\theta = \pi, \frac{7\pi}{4}}$

4.1, 6.3 (ii) using the graph given, Find the values of θ between 0° and 360° that satisfy the following. Show clearly how you obtained your answers and Label the graph.

(a) $\tan\theta = 4$ From graph $\boxed{\theta = 76^\circ, 256^\circ}$

(b) $\tan\theta = -6$ ~ ~ $\boxed{\theta = 99^\circ, 279^\circ}$

$\boxed{\theta = 99^\circ, 279^\circ}$

(c) $\tan 3\theta = 2 \Rightarrow \boxed{3\theta = 63, 243, 423, 603, 783, 963}$

$\boxed{\theta = 21^\circ, 81^\circ, 141^\circ, 201^\circ, 261^\circ, 321^\circ}$

(d) $\tan(\theta - 60) = -1 \Rightarrow \boxed{(\theta - 60) = 135^\circ, 315^\circ}$

$\boxed{\theta = 195^\circ, 375^\circ}$

5.1 using the formulas, solve the following, for $-90 < \theta \leq 90^\circ$

(a) $1 - \sin^2\theta = 0.5 \Rightarrow \sin^2\theta = 0.5 \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\boxed{\therefore \theta = 45^\circ}$

(b) $\sin(60 + \theta) = \cos\theta$

$\sin 60 \cos\theta + \cos 60 \sin\theta = \cos\theta$

$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \cos\theta$

$\therefore \frac{0.5\sin\theta}{\cos\theta} = \frac{0.134\cancel{\cos\theta}}{\cancel{\cos\theta}} \Rightarrow 0.5\tan\theta = 0.134$

$\tan\theta = \frac{0.134}{0.5} = 0.268$

$\boxed{\theta = \tan^{-1}(0.268) = 15^\circ}$

$\boxed{\text{to check}}$

ⓐ $1 - \sin^2 45 = 1 - \left(\frac{1}{\sqrt{2}}\right)^2 = 1 - \frac{1}{2} = 0.5$ ✓

ⓑ $\sin(60+15) = \sin 75 = 0.966$ ✓

$\cos(15) = 0.966$ ✓

ⓒ $(\cos(\theta) + \sin(\theta))^2 = 1.5$

$\therefore \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 1.5$

$\therefore 1 + \sin 2\theta = 1.5 \Rightarrow \boxed{\sin 2\theta = 1.5 - 1 = 0.5}$

$\therefore 2\theta = \sin^{-1}(0.5) \Rightarrow 2\theta = 30^\circ \Rightarrow \theta = \frac{30^\circ}{2}$

$\boxed{\theta = 15^\circ}$

to check $(\cos(15) + \sin(15))^2 = 1.5$ ✓

ⓓ $2\cos(\theta+30)\cos(\theta-30) = 1$ (factor formula)

$2[\cos\theta \times \cos 30 - \sin\theta \sin 30][\cos\theta\cos 30 + \sin\theta \sin 30] = 1$

$2[0.866\cos\theta - 0.5\sin\theta][0.866\cos\theta + 0.5\sin\theta] = 1$

$2[\frac{3}{4}\cos^2\theta + \cancel{0.433\sin\theta\cos\theta} - \cancel{0.433\sin\theta\cos\theta}$

$- 0.25\sin^2\theta] = 1$

$2[\frac{3}{4}\cos^2\theta - \frac{1}{4}\sin^2\theta] = 1$

$\therefore [\frac{3}{4}\cos^2\theta - \frac{1}{4}\sin^2\theta] = \frac{1}{2}$

$\therefore \cancel{\frac{1}{4}[}\ \cancel{\frac{3}{4}[\cos^2\theta - \frac{1}{3}}$

$$\left[\frac{3}{4}\cos^2\theta - \frac{1}{4}(1-\cos^2\theta)\right] = \frac{1}{2}$$

$$\therefore \frac{3}{4}\cos^2\theta - \frac{1}{4} + \frac{1}{4}\cos^2\theta = \frac{1}{2}$$

$$\therefore \cos^2\theta = \frac{3}{4} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \cos^{-1}\frac{\sqrt{3}}{2}$$

$$\boxed{\therefore \theta = 30^\circ}$$

To check $2\cos(30+30)\cos(30-30)$

$= 2\cos 60 \cos 0 = 2 \times 0.5 \times 1 = 1$

5.2 use the formulas you know to prove the following identities;

(a) $(\sin A + \cos A)(\sin B + \cos B) = \sin(A+B) + \cos(A-B)$

$\sin A \sin B + \sin A \cos B + \cos A \sin B + \cos A \cos B \longrightarrow (1)$

$\sin(A+B) + \cos(A-B) = \sin A \cos B + \cos A \sin B + \cos A \cos B$
$+ \sin A \sin B \longrightarrow (2)$

$$\boxed{\therefore (\sin A + \cos A)(\sin B + \cos B) = \sin(A+B) + \cos(A-B)}$$

(b) $\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} = \cancel{2\operatorname{cosec} 2\theta}$

we have, $\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$

$2\operatorname{cosec} 2\theta = \dfrac{2}{\sin 2\theta} = \dfrac{\cancel{2}}{\cancel{2}\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$

$$\boxed{\therefore \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = 2\operatorname{cosec} 2\theta}$$

$$\frac{\sin 3\theta + \sin\theta}{\sin 5\theta + \sin 3\theta} = \frac{\sin 2\theta}{\sin 4\theta} \quad \text{(Factor Formula)}$$

Sol

We have, $\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Applying this to the numerator $\sin 3\theta + \sin\theta$ with $A = 3\theta$ and $B = \theta$ gives:

$$\sin(3\theta) + \sin\theta = 2\sin\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)$$

$$\boxed{\therefore \sin(3\theta) + \sin\theta = 2\sin 2\theta\cos\theta}$$

We have, $\sin 5\theta + \sin 3\theta = 2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)$

$$\boxed{\therefore \sin 5\theta + \sin 3\theta = 2\sin 4\theta\cos\theta}$$

$$\therefore \frac{\sin 3\theta + \sin\theta}{\sin 5\theta + \sin 3\theta} = \frac{\cancel{2}\sin 2\theta\cancel{\cos\theta}}{\cancel{2}\sin 4\theta\cancel{\cos\theta}} = \frac{\sin 2\theta}{\sin 4\theta}$$

$$\boxed{\therefore \frac{\sin 3\theta + \sin\theta}{\sin 5\theta + \sin 3\theta} = \frac{\sin 2\theta}{\sin 4\theta}}$$

$\boxed{5.3, 5.4, 5.5}$

(a) Write $4\cos(x) + 3\sin(x)$ in the form $R\cos(x+\phi)$, giving ϕ to within $0.1°$.

Sol

$R = \sqrt{4^2+3^2} = \sqrt{25} \Rightarrow \boxed{R = 5}$

But, $R\cos(X+\phi) = R\cos X\cos\phi - R\sin X\sin\phi$

$\therefore R\cos\phi = 4$, $-R\sin\phi = 3 \Rightarrow R\sin\phi = -3$

$\cos\phi = \frac{4}{R}$, $\sin\phi = \frac{-3}{R}$ $\Rightarrow \phi$ Lies in quadrant (4)

$\frac{\sin\phi}{\cos\phi} = \frac{-3}{\cancel{R}} * \frac{\cancel{R}}{4} \Rightarrow \boxed{\tan\phi = \frac{-3}{4}}$

$\therefore \phi = \tan^{-1}\left(\frac{-3}{4}\right) \Rightarrow \boxed{\phi = -36.9^\circ}$

$$\boxed{\therefore 4\cos(X) + 3\sin(X) = 5\cos(X - 36.9)}$$

(b) Hence solve $4\cos(X) + 3\sin(X) = 3.5$ for $-18^\circ < X < 18^\circ$

sol

we have from the previous problem.

$4\cos(X) + 3\sin(X) = R\cos(X+\phi)$

$R = \sqrt{4^2+3^2} = \sqrt{25} = 5 \Rightarrow \boxed{R = 5}$

$\therefore 4\cos(X) + 3\sin(X) = R\cos(X+\phi)$

$\therefore 4\cos(X) + 3\sin(X) = R\cos(X)\cos(\phi) - R\sin(X)\sin(\phi)$

$\therefore R\cos(\phi) = 4$, $-R\sin(\phi) = 3 \Rightarrow R\sin(\phi) = -3$

$\therefore \frac{R\sin(\phi)}{R\cos(\phi)} = \frac{-3}{4} \Rightarrow \phi = \tan^{-1}\left(\frac{-3}{4}\right) \boxed{\phi = -36.9^\circ}$

$\therefore 5\cos(X - 36.9) = 3.5 \Rightarrow \cos(X - 36.9) = \frac{3.5}{5} = 0.7$

$\therefore (X - 36.9) = \cos^{-1}(0.7) = 45.57^\circ \Rightarrow \boxed{X = 82.47^\circ}$

(c) state the maximum and minimum values of $4\cos(x)+3\sin(x)$ and where they occur within the range. $-180 < x < 180°$.

Sol

we have, $4\cos(x)+3\sin(x) = 5\cos(x-36.9)$ proved previously

Maximum value $= 5$, Minimum value $= -5$

Maximum

$5\cos(x-36.9) = 5$

$\therefore \cos(x-36.9) = 1$

$\therefore (x-36.9) = \cos^{-1} 1$

$\therefore (x-36.9) = 0$

$\boxed{\therefore x = 36.9°}$

Minimum

$5\cos(x-36.9) = -5$

$\therefore \cos(x-36.9) = -1$

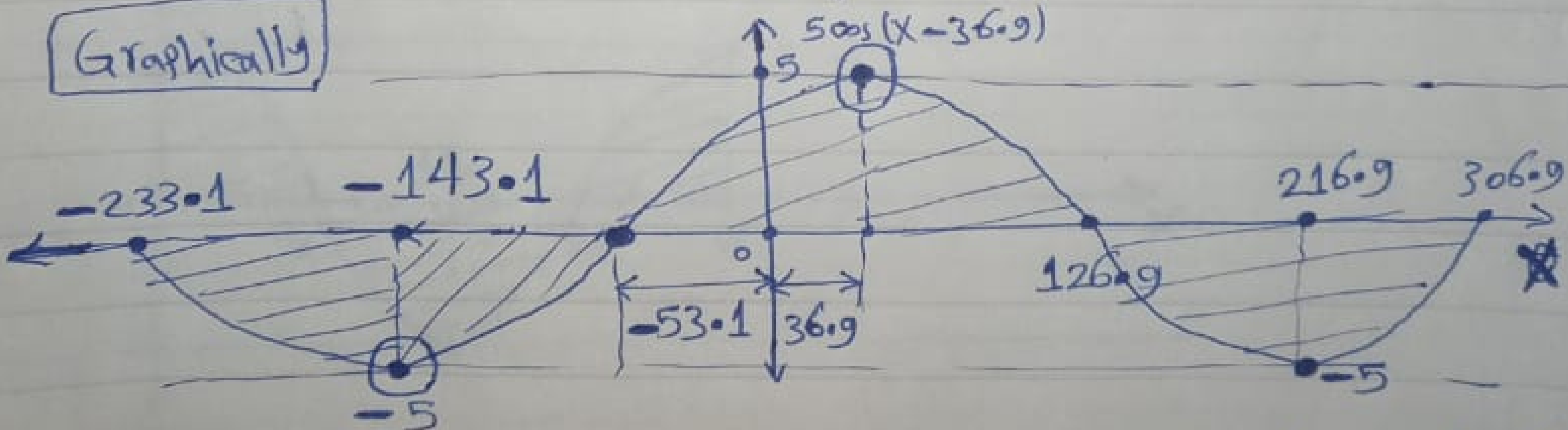
$\therefore (x-36.9) = \cos^{-1}(-1)$

$\therefore (x-36.9) = -180°$

$\therefore x = -180+36.9$

$\boxed{x = -143.1°}$

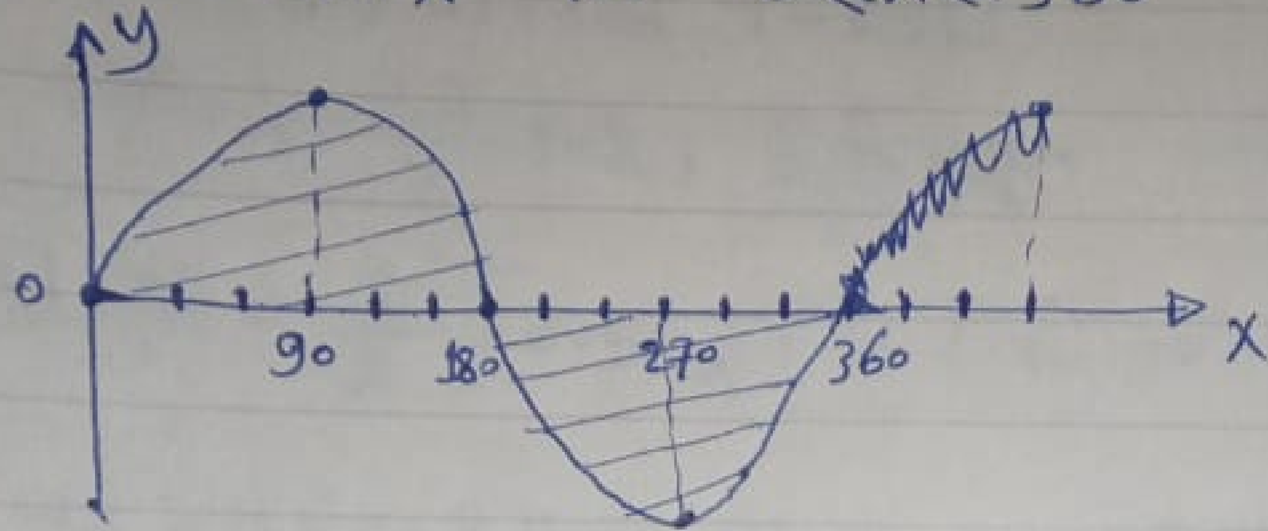
Graphically



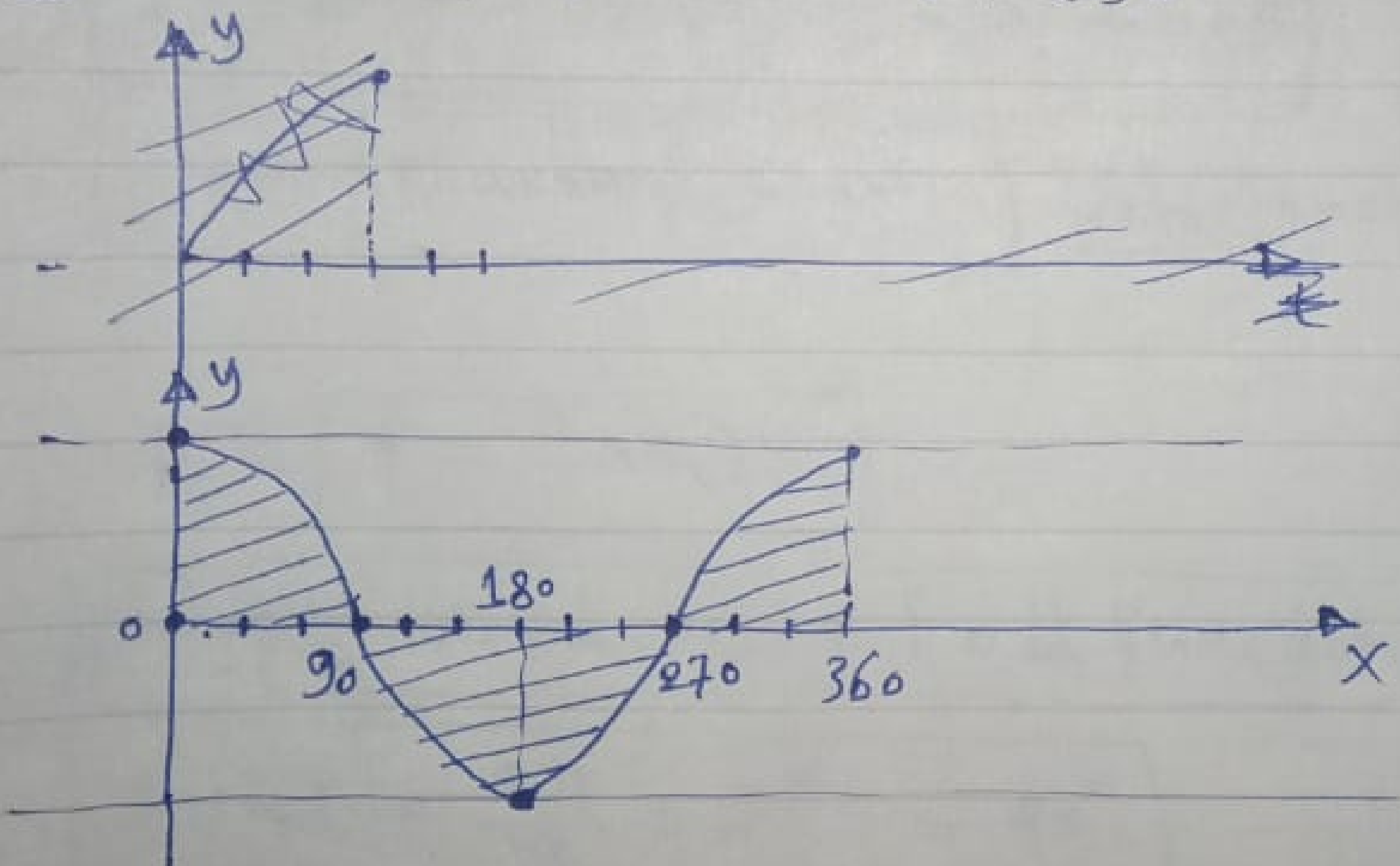
for $-180 < x < 180$ — Maximum at $x = 36.9°$ — Minimum at $x = -143.1$

6.1 Sketch the following graphs. Label the axes.

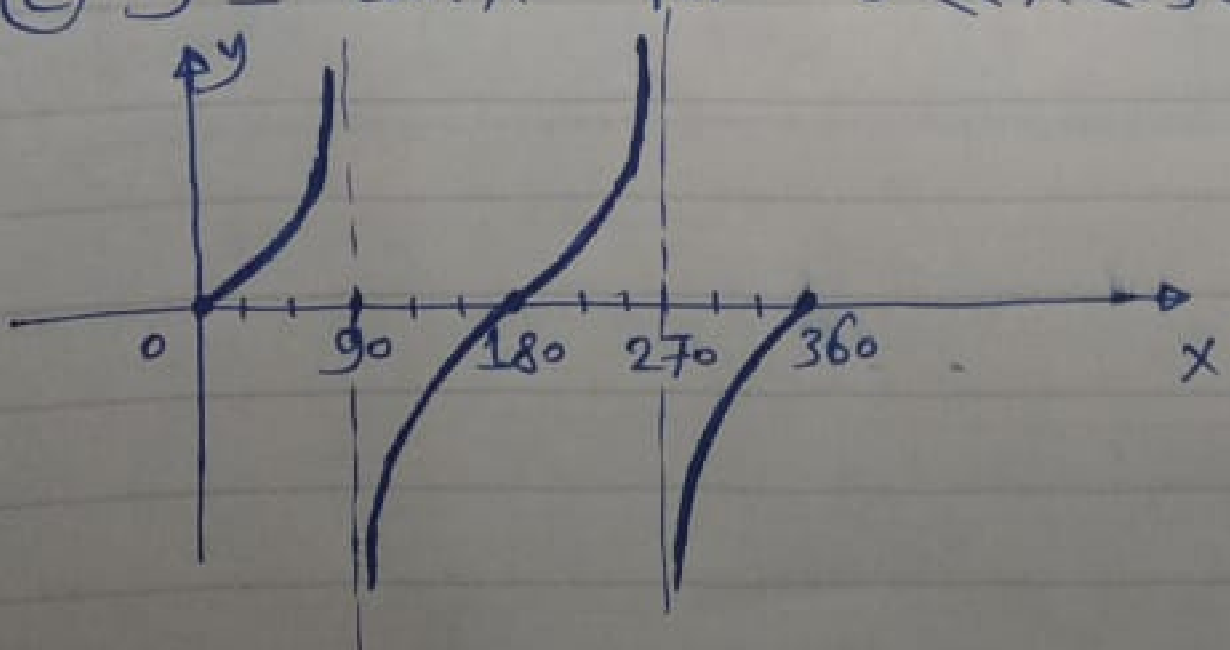
(a) $y = \sin x$ for $0 \leq x \leq 360°$



(b) $y = \cos x$ for $0 \leq x \leq 360$



(c) $y = \tan x$ for $0 \leq x \leq 360$



6.2 (a) $\boxed{y = 2\sin(3\theta - 90^\circ) + 3}$

symmetry around y-axis

Period $\frac{2\pi}{3}$ rad,

(b) $y_P = \frac{1}{2} = 0.5$, $\boxed{T = 3.2}$ $\Rightarrow f = \frac{1}{T} = \frac{1}{3.2} = 0.31\ Hz$

$w = 2\pi f = 2\pi \times 0.31 = 0.625\pi$ rad/s

$\boxed{y(t) = 0.5\sin\left(\frac{5\pi}{8}t - 90\right) + 0.5}$

$\boxed{\text{symmetry around y axis}}$ $\boxed{\text{even Function}}$

(c) $\boxed{y = \tan(9x)}$

$\boxed{\text{symmetry around origin (odd function)}}$

$\boxed{\text{period} = \frac{2\pi}{9} = \frac{360}{9} = 40^\circ}$