Probability

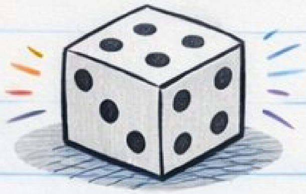
Part 1: Introduction

- **Probability** = Chance of an event happening.

  It tells us **how likely** or **unlikely** something is to happen.

Examples:

Coin Toss — Rolling a Dice — Picking a Card



Used in: Math, Science, Games, Weather, Statistics

Important Terms:

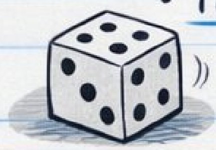
1. Experiment:

An action whose result cannot be predicted with certainty.

Examples:

- Tossing a Coin

- Throwing a Dice



- **2. Outcome:**

  Possible result of an experiment.

  Example:

  - Coin Toss:
    - Head (H)
    - Tail (T)

  - Sample Space:

    For a Coin Toss:

    $S = \{H, T\}$

    For a Dice:

    $S = \{1, 2, 3, 4, 5, 6\}$

Probability -
- Part 2 -

Types of Events

1. Complementary Events

   Example: ______

2. Mutually Exclusive Events

   Example: ______

3. Exhaustive Events

   Example: S = ...

Complementary Events

If A is an event, then

$$\bar{A} + \bar{A} = S$$

$$P(A) + P(A') = 1$$

Mutually Exclusive Events

Events that cannot occur together

$$A \cap B = \varnothing$$

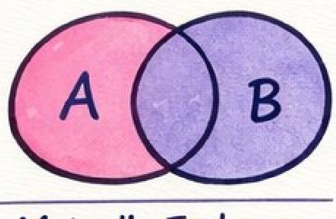
$$P(A \cup B) = P(A) + P(B)$$

Exhaustive Events

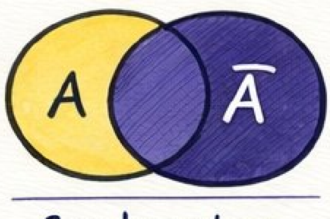
Events that cover the entire Sample Space

$$A_1 + A_2 + A_3 + ... = S$$

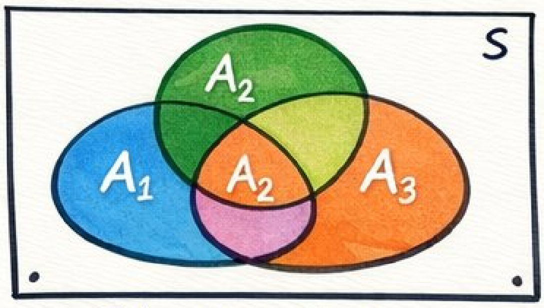
$$P(A_1) + P(A_2) + P(A_3) + ... = 1$$



Mutually Exclusive



Complementary



Exhaustive Events

More to Come...

Part 3: Probability

(Formulas & Examples)

Probability Formula.

Probability of an Event (P(E) = $\frac{\text{No.of Favourable Outcomes}}{\text{Total No.of Possible Outcomes}}$

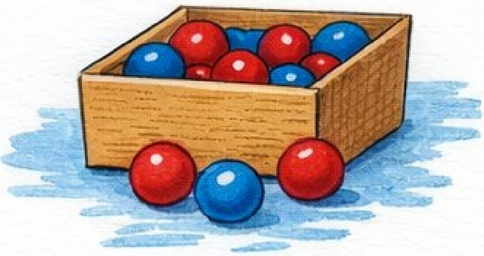
$$P(E) = \frac{n(E)}{n(S)}$$

where: $n(E)$ = Number of Favourable Outcomes

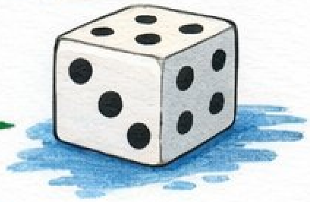
$n(S)$ = Total Number of Possible Outcomes

Important Points:

1. Probability of an Event $(0 \leq P(E) \leq 1)$
2. Probability of Sure Event $P(E) = 1$ (Certain)
3. Probability of Impossible Event $P(E) = 0$ (Impossible)



- Probability of 'NOT E' = $1 - P(E)$
- $P(E) + P(\text{Not } E) = 1$

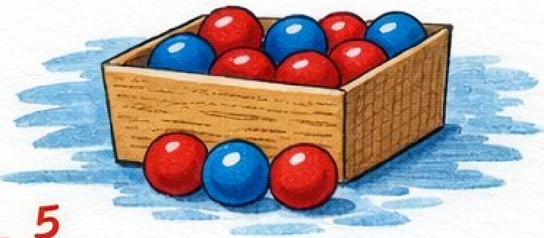


Solved Examples:

1. A box contains 5 Red and 3 Blue balls. One ball is chosen at random. Find the probability of getting a Red ball.

Solution:

- Total Balls, $n(S) = 5 + 3 = 8$
- Favourable Outcomes of Red balls, $n(E) = 5$



$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{5}{8} = \frac{5}{8}$ → P(Red ball) = $\frac{5}{8}$

2. A dice is thrown. Find the probability of getting a number greater than 4.

Solution:

- Favourable Outcomes = $\{5, 6\}$ → $P(E) = \frac{2}{n(S)} = \frac{1}{3}$

→ $P(\text{Number} > 4) = \frac{1}{3}$

⭐ Probability (Part-4) ⭐

1. Probability of "Not" Event

If A is an event, then the probability of "Not A" ($\bar{A}$) is:

$$P(\bar{A}) = 1 - P(A)$$

Ex: If $P(A) = 0.7$, then $P(\bar{A}) = 1 - 0.7 = 0.3$.

$P(\bar{A}) = 1 - P(A)$ → Prob. of Not A → Prob. of A

2. At Least / At Most Type Questions

→ "At Least 1" = $1 - P(\text{None})$

→ "At Most n" = $P(0) + P(1) + \ldots + P(n)$

Ex: Probability of getting at least 1 six in 4 dice throws?

$$P(\text{At Least 1 six}) = 1 - P(\text{No sixes}) = 1 - \left(\frac{5}{6}\right)^4 = 0.804$$

3. Expected Value

$$\text{Expected Value } (E) = \sum x_i \cdot P(x_i)$$

Where x_i = Value of outcome

$P(x_i)$ = Probability of that outcome.

Ex: In a game, winnings are:

- Rs 10 with $P = 0.2$
- Rs 20 with $P = 0.5$
- Rs 0 with $P = 0.3$

$$E = 10 \times 0.2 + 20 \times 0.5 + 0 \times 0.3 = 14$$

4. Summary of Probability

1. Probability is always between 0 and 1
2. $P(\text{Sample Space}) = 1$

3. $P(A) + P(A') = 1$
4. Impossible Event: $P(\phi) = 0$

4. Certain Event: $P(S) = 1$

Certain

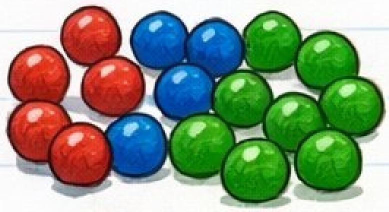
Probability: Part 5

IMPORTANT QUESTIONS

Exam-Level Questions

Q1. A bag contains 5 red, 7 blue and 8 green. balls.

(a) Find the probability of drawing a red ball.



Total = 20

Ans:

$$P(\text{Red}) = \frac{\text{No. of Red Balls}}{\text{Total No. of Balls}} = \frac{5}{20} = \frac{1}{4}$$

$$= \frac{5}{20}$$

$$= \underline{20}$$

(b) Find the probability of drawing a blue or a green ball.

Ans:

$$P(\text{Blue or Green}) = \frac{7+8}{20} = \frac{15}{20}$$

$$= \frac{15}{20}$$

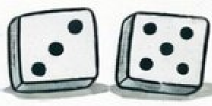
Q2. A die is thrown. Find the probability of getting:

(a) An even number.

(b) A prime number.

(c) A number > 4.

Ans: $P(\text{Even}) =$  $= \frac{1}{2}$



Q3. Two coins are tossed. Find the probability of:

(a) Getting two Heads.

Ans: $P(\text{Two Heads}) = \boxed{HH} = \frac{1}{4}$

(b) $P(\text{At least 1 Tail}) = \boxed{HT,}\ \boxed{TH,}\ \boxed{TT} = \frac{3}{4}$

Probability
Part – 6
(MCQ + Assertion – Reason)

Multiple Choice Questions (MCQs)

Choose the correct answer:

1) The probability of an impossible event is :

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

2) A dice is rolled. The probability of getting an even number is :

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{6}$

3) The probability of sure event is :

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $3/4$

Assertion – Reason Questions

Choose the correct option:

(A) Both Assertion and Reason are true & Reason is the correct explanation of Assertion.

(B) Both Assertion and Reason are true but Reason is not correct explanation of Assertion.

(C) Assertion is true but Reason is false.

(D) Assertion is false but Reason is true.

1. Assertion: "Sum of probablities of all outcomes of an experiment is 1."
   Reason: "Probability of an event can never be greater than 1."

2. Assertion: "A deck of cards has 26 red cards."
   Reason: "There are 4 Queens in a deck of cards."