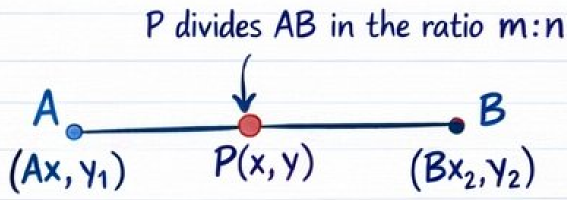
PART 2 : SECTION FORMULA

Section Formula : If a Point $P(x, y)$ divides the line segment AB joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m:n$, then

$$P(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$



Example 1 : Find the coordinates of the point P which divides the line segment joining $A(2,3)$ and $B(8,7)$ internally in the ratio $3:2$.

Solution.:

Here, $A(x_1, y_1) = (2,3)$, $B(x_2, y_2) = (8,7)$, Ratio = $m:n = 3:2$ ($m = 3$, $n = 2$)

Using Section Formula,

$$P(x,y) = \left(\frac{3\times 8 + 2\times 2}{3+2}, \left(\frac{3\times 7 + 2\times 3}{3+2}\right)\right)$$

$$= \left(\frac{24+4}{5}, \frac{21+6}{5}\right)$$

$$= \left(\frac{28}{5}, \frac{27}{5}\right)$$

$$= (5.6, 5.4)$$

∴ The coordinates of P are $(5.6, 5.4)$.

Example 2: Show that the point $C(4, -1)$ divides the line segment joining $A(1, -3)$ and $B(6, 2)$ in the ratio $2:3$.

Solution:

Let $C(x, y) = (4, -1)$,

Here, $A(x_2\ y_1) = (1, -3)$, $B(x_2, y_2) = (6, 2)$.

Ratio = $m:n = 2:3$

Using Section Formula, $C(x, y) = \left(\frac{2\times 6 + 3\times 1}{2+3}\right), \left(\frac{2\times 2 + 3\times (-3)}{2+3}\right)$

$$= \left(\frac{12+3}{5}, \frac{4-9}{5}\right)$$

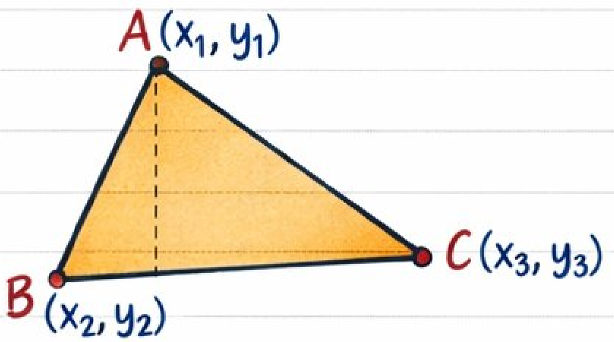
$$= (3, -1)$$

∴ $C(4, -1)$ divides AB in the ratio $2:3$.

PART 3: AREA OF TRIANGLE

$$\text{Area of } \Delta = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

where $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle.



Q. Find the area of the triangle whose vertices are $A(2,4)$, $B(6,8)$ and $C(4,2)$.

Solution:

1. $x_1 = 2$, $y_1 = 4$; $x_2 = 6$, $y_2 = 8$; $x_3 = 4$, $y_3 = 2$;
2. Using the formula:

$$\text{Area of } \Delta = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

3. Substitute the values:

$$= \frac{1}{2}\left| 2(8-2) + 6(2-4) + 4(4-8) \right|$$

$$= \frac{1}{2}\left| 2 \times 6 + 6 \times (-2) + 4 \times (-4) \right|$$

$$= \frac{1}{2}\left| 12 - 12 - 16 \right|$$

$$= \frac{1}{2}\left| -16 \right|$$

$$= \frac{1}{2} \times 16$$

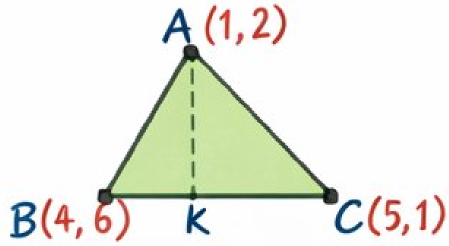
$$= 8 \text{ sq. units}$$

$\therefore$ Area of the triangle $= 8$ square units.

PART 4: Important Board Questions

Q.1. Find the area of triangle whose vertices are $A(1, 2)$, $B(4, 6)$ & $C(5, 1)$.

Solution:



$A(1,2)$ $B(4,6)$ $C(5,1)$

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|1(6-1) + 4(1-2) + 5(2-6)\right|$$

$$= \frac{1}{2}\left|5 - 4 - 20\right|$$

$$= \frac{1}{2}\left| \times 19\right.$$

$\therefore$ Area = 9.5 sq. units.

Q.2. Find the value of k if area of triangle is 24 sq. units with vertices $(2, 3)$, $(k, 7)$ & $(4, 3)$.

Solution:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= 24 = \frac{1}{2}\left|2(7-3) + k(3-3) + 4(3-7)\right|$$

$$= 24 = \frac{1}{2}\left|8 - 0 - 4k\right|$$

$$= 24 = \left|4 - 4k\right|$$

$$= 24 = 4(1-k)$$

$$\Rightarrow 1 - k = 6$$

$$k = -5$$

$\therefore k = -5$

Q.3. Prove that $(4, 1)$, $(-2, 3)$ & $(1, 5)$ are collinear.

Solution:

$$\text{Area of } \Delta ABC = \frac{1}{2}\left|x_2(y_2 - y_3) + x_3(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|4(3-5) + (-2)(5-1) + 1(1-3)\right|$$

$$= \frac{1}{2}\left|-8 + 8 - 2\right|$$

$$= \frac{1}{2} \times 0$$

$$= 0$$

PART 5: EXTRA PRACTICE QUESTIONS

1) Find the distance between (3, 4) and (−1, −2):

$$\text{Distance} = \sqrt{x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1-3)^2 + (-2-4)^2}$$
$$= \sqrt{(-4)^2 + (-6)^2}$$
$$= \sqrt{16 + 36}$$
$$= \sqrt{52}$$

Ans: $2\sqrt{13}$ units

2.) Find the midpoint of A(6, 8) and B(−4, 2):

$$\text{Midpoint} = \left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$$
$$= \left(\frac{6 + (-4)}{2}, \frac{8 + 2}{2}\right)$$
$$= \left(\frac{2}{2}, \frac{10}{2}\right)$$
$$= (1, 5)$$

Ans: (1, 5)

3.) Check if the points (2, −1), (0, 3), and (−2, 7) are collinear.

$$\text{Slope of AB} = \frac{3+1}{0-2} = \frac{4}{-2} = -2$$
$$\text{Slope of BC} = \frac{7-3}{-2-0} = \frac{4}{-2} = -2$$

Since Slopes of AB & BC are equal,

∴ Points are Collinear.

Ans: Collinear

4.) Find the area of the triangle with vertices P(1, 2), Q(4, 6), R(6, 2).

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_2) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$
$$= \frac{1}{2}\left|1(6-2) + 4(2-2) + 6(2-6)\right|$$
$$= \frac{1}{2}\left|4 + 0 - 24\right|$$
$$= \frac{1}{2}\left|-20\right|$$

Ans: 10 sq. units

PART 5: EXTRA PRACTICE QUESTIONS – SOLUTIONS

1. Distance Formula:

Find the distance between $A(3, 4)$ and $B(-1, 1)$.

Solution:

$$\text{Distance } AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(-1-3)^2 + (1-4)^2}$$
$$= \sqrt{(-4)^2 + (-3)^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$

∴ Distance AB = **5 units.**

2. Midpoint Formula:

Find the midpoint of $P(2, 6)$ and $Q(-4, 2)$.

Solution:

$$\text{Midpoint } M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{2-4}{2}\right), \frac{6+2}{2}\right)$$
$$= \left(\frac{-2}{2}, \frac{8}{2}\right)$$
$$= (-1, 4)$$

∴ Midpoint M = **(−1, 4).**

3. Collinearity Check:

Check if $A(1, 2)$, $B(4, 5)$, and $C(7, 8)$ are collinear.

Solution:

$$\text{Slope of } AB = \frac{y_2-y_1}{x_3-x_1} = \frac{5-2}{4-1}$$
$$= \frac{3}{3} = \frac{1}{3}$$

$$\text{Slope of } BC = \frac{y_2-y_2}{x_3-x_2} = \frac{8-5}{7-4}$$
$$= \frac{3}{3} = \frac{1}{3}$$

Since Slope of AB = Slope of BC,

∴ Points **A, B and C** are collinear.

4. Area of Triangle:

Find the area of triangle with vertices $X(1, 1)$, $Y(4, 3)$, $Z(3, 7)$.

Solution:

$$\text{Area} = \frac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$$
$$= \frac{1}{2}|1(3-7) + 4(7-1) + 3(1-3)|$$
$$= \frac{1}{2}|-4 + 24 - 6|$$
$$= \frac{1}{2}|14|$$
$$= 7$$

∴ Area = **7 square units.**

PART 5 : EXTRA PRACTICE QUESTIONS

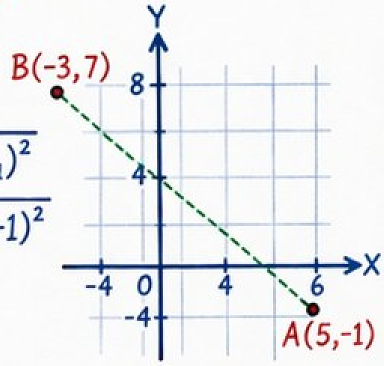
(Coordinate Geometry) – More Questions with Solutions

① Find the distance between A(5, −1) and B(−3, 7).

Solution:

Using Distance Formula,

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-3-5)^2+(7-(-1)^2}$$
$$= \sqrt{(-8)^2+(8)^2}$$
$$= \sqrt{64+64}$$
$$= \sqrt{128}$$
$$= 8\sqrt{2} \text{ units}$$



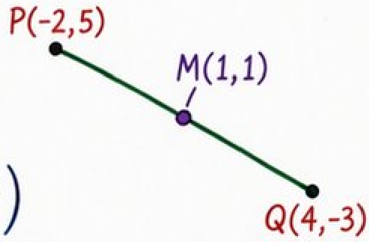
Ans: $8\sqrt{2}$ units

② Find the coordinates of the midpoint of the line segment joining P(−2, 5) and Q(4, −3).

Solution:

Midpoint Formula,

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{-2+4}{2}, \frac{5+(-3)}{2}\right)$$
$$= \left(\frac{2}{2}, \frac{2}{2}\right)$$
$$= (1, 1)$$



Ans: (1, 1)

③ Check whether the points A(1, 2), B(2, 4), C(3, 6) are collinear.

Solution:

For collinearity, Area of $\Delta ABC = 0$

$$\text{Area} = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$
$$= \frac{1}{2}\left|1(4-6)+2(6-2)+3(2-4)\right|$$
$$= \frac{1}{2}\left|1(-2)+2(4)+3(-2)\right|$$
$$= \frac{1}{2}\left|-2+8-6\right|$$
$$= \frac{1}{2}\left|0\right| = 0$$

Ans: Yes, the points are collinear.

④ Find the value of k if the point (k, 2k) divides the line segment joining A(2, −3) and B(8, 5) in the ratio 1 : 2 internally.

Solution:

Using Section Formula (Internal Division),

$$(k, 2k) = \left(\frac{1\times 8+2\times 2}{1+2}, \frac{1\times 5+2\times(-3)}{1+2}\right)$$
$$= \left(\frac{8+4}{3}, \frac{5-6}{3}\right)$$
$$= \left(\frac{12}{3}, \frac{-1}{3}\right)$$
$$= \left(4, -\frac{1}{3}\right)$$

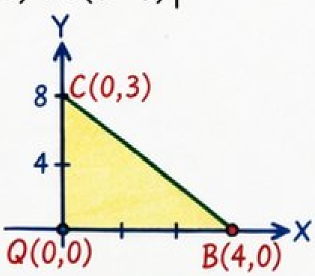
So, $k = 4$ and $2k = -\frac{1}{3} \Rightarrow k = -\frac{1}{6}$

Ans: $k = -\frac{1}{6}$

⑤ Find the area of the triangle formed by the points A(0, 0), B(4, 0) and C(0, 3).

Solution:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$
$$= \frac{1}{2}\left|0(0-3)+4(3-0)+0(0-0)\right|$$
$$= \frac{1}{2}\left|0+12+0\right|$$
$$= \frac{1}{2}\left|12\right|$$
$$= 6 \text{ sq. units}$$



Ans: 6 sq. units

⑥ If A(3, −1), B(6, k) and C(9, 3) are collinear, find the value of k.

Solution:

Area of $\Delta ABC = 0$

$$\frac{1}{2}\left|3(k-3)+6(3-(-1))+9(-1-k)\right| = 0$$
$$\frac{1}{2}\left|3k-9+24-9-9k\right| = 0$$
$$\frac{1}{2}\left|-6k+6\right| = 0$$
$$\left|-6k+6\right| = 0$$
$$-6k+6 = 0 \Rightarrow k = 1$$

Ans: k = 1

★ Practice Makes Perfect! ★

Keep Practicing and Score Well in Exams!

PART 5 : EXTRA PRACTICE Q

1. Distance Formula

Find the distance between A(1, 2) and B(5, 6).

Solution:

$$\text{Distance } AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(5-1)^2 + (6-2)^2}$$
$$= \sqrt{4^2 + 4^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32} = 4\sqrt{2}$$

Distance AB = $4\sqrt{2}$ units.

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3. Collinearity Check

Are the points A(2, 3), B(4, 7), C(6, 11) collinear?

Solution:

$$\text{Slope of } AB = \frac{y_2-y_1}{x_2-x_1} = \frac{7-3}{4-2} = \frac{4}{2} = 2$$

$$\text{Slope of } BC = \frac{11-7}{x_2-x_1} = \frac{6-4}{4-2} = \frac{4}{2} = 2$$

Slopes are equal $\Rightarrow$ Points are Collinear

Yes, A, B, C are collinear.

QUESTIONS — SOLUTIONS

2. Midpoint Question

Find the midpoint of $P(3, 8)$ and $Q(7, 4)$.

Solution:

$$\text{Midpoint } M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{3+7}{2}, \frac{8+4}{2}\right)$$

$$= \left(\frac{10}{2}, \frac{12}{2}\right)$$

$$= (5, 6)$$

$$\text{Midpoint } M = (5, 6)$$

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4. Area of Triangle

Find the area of $\triangle PQR$ with $P(1, 1)$, $Q(4, 5)$, $R(6, 2)$.

Solution:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|1(5-2) + 4(2-1) + 6(1-5)\right|$$

$$= \frac{1}{2}\left|3 + 4 - 24\right|$$

$$= \frac{1}{2}\left|-17\right|$$

$$= \frac{2}{4}\,2$$

$$\text{Area} = 8.5 \text{ sq. units.}$$