Surds

Roots and powers are closely related, but only some roots can be written as whole numbers. Surds are roots which cannot be written in this way. Nevertheless, it is possible to manipulate surds, and to simplify formulæ involving them.

Exercise 1: Identify surds or not

number	equals	Surd or not
$\sqrt{1}$		
$\sqrt{2}$		
$\sqrt{3}$		
$\sqrt{4}$		
$\sqrt{5}$		
$\sqrt{6}$		
$\sqrt{7}$		
$\sqrt{8}$		
$\sqrt{9}$		
$\sqrt{10}$		
$\sqrt{12}$		
$\sqrt{13}$		

√14		
√15		
√16		
√17		
√18		
√20		
√21		
√22		
√23		
√24		
√25		

Rule 1: $\sqrt{a} \times \sqrt{a} = a$

Example 1: $\sqrt{2} \times \sqrt{2} = 2$

Example 2: $\sqrt{2.5} \times \sqrt{2.5} = 2.5$

Exercise 2:

practice the multiples of same surds

Multiples	Answer
$\sqrt{2} \times \sqrt{2}$	2
$\sqrt{3} \times \sqrt{3}$	
$\sqrt{4} \times \sqrt{4}$	4
$\sqrt{5} \times \sqrt{5}$	
$\sqrt{6} \times \sqrt{6}$	
$\sqrt{7} \times \sqrt{7}$	
$\sqrt{8} \times \sqrt{8}$	
$\sqrt{9} \times \sqrt{9}$	

Rewriting surds

Example 1: how do you rewrite $\sqrt{20}$?

Step 1: factorise it

$$\sqrt{2x2x5}$$

Step 2: find the numbers that repeats twice (a pair), take one outside as a full number. The rest is left in the square root

In other words, imagine you have people live in a house (√). You will only pick an identical couple out, and they is call 1 pair.
√ **2**x**2**x5 = take only <u>one</u> = **2**√ **5**

-In this house there are 3 people, but we can only make 1 identical couple, 2 and 2
Step 3: take the 'pair' out of the house (√).

-We take this said couple out of the house (√). <u>But</u> we can only write about them once, **so we write**
(1) one 2 outside the house (√)
(2) inside the house (√), 5 is left

Hence the expression √ 20 = √ 2x2x5 = 2√ 5

Example 2: how do you rewrite √ 80? (There is more than one pair)

Step 1: factorise it

√ 2x2x2x2x5

Step 2: find the pairs

√ 2x2x2x2x5

There are two pairs, 2 and 2

Step 3: Take the pair out

(1) Outside the house (√), we write 2x2 = 4

(2) Inside the house (√), 5 is left

Hence the expression √ 80 = √ 2x2x2x2x5

= 2x2√ 5 = 4√ 5

Example 3: how do you rewrite √ 160? (Lets try to work faster by not factoring with prime number)

Step 1: factorise it (This time we want to work faster, because we know 4x4 =16)

√ 160 = √ 4 x4 x 10

Step 2: find the pair

√ 160 = √ 4 x4 x 10

Step 3: write the pair outside the house (√)

4 √10

Exercise 3: rewrite or simplify surds

surd	factorised	Take the pair out	rewrite
√ 50	√ **5 x 5**x2	**5 and √2**	5√2
√70			

√90			
√120			
√180			
√200			
√360			
√1040			
√2500			

Rule 2: subtraction and addition of number with the same surd

$$a\sqrt{b} + c\sqrt{b} = a \times c \sqrt{b}$$

Example 1: try $5\sqrt{20} + 7\sqrt{20}$

$$5\sqrt{20} + 7\sqrt{20} = 5 + 7\ \sqrt{20}$$
$$= 13\sqrt{20}$$

Example 2: try $9\sqrt{1999} + 1\sqrt{1999}$

$$9\sqrt{1999} + 1\sqrt{1999} = 9+1\ \sqrt{1999}$$
$$= 10\ \sqrt{1999}$$

Exercise 4: addition and subtraction of same surd

$5\sqrt{2}+4\sqrt{2}=$

$4\sqrt{5} + 3\sqrt{5} =$

$2\sqrt{10} - 4\sqrt{10} =$

$3\sqrt{7} - 3\sqrt{7} =$

$5\sqrt{13} + 1\sqrt{13} =$

$2\sqrt{7} - 2\sqrt{7} =$

$5\sqrt{2} - 1\sqrt{2} =$

$2\sqrt{6} - 6\sqrt{6} =$

$2\sqrt{10} - 6\sqrt{10} =$

$5\sqrt{13} + 1\sqrt{13} =$

$5\sqrt{2} + 6\sqrt{2}=$

$5\sqrt{3} - 5\sqrt{3}=$

$3\sqrt{2} - 4\sqrt{2}=$

Rule 3

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$
$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, .$$

Example: $\sqrt{30} = \sqrt{5} \times 6$

According to this rule $\sqrt{30} = \sqrt{5} \times \sqrt{\mathbf{6}}$

But you can factorise 6 further **hence** $\sqrt{\mathbf{6}} = \sqrt{\mathbf{2 \times 3}}$

Then you can rewrite the whole expression

$$\sqrt{30} = \sqrt{5} \times \sqrt{2} \times \sqrt{3}$$

When will this be useful?

When you have to add numbers of different surds like

$\sqrt{32} + \sqrt{18} = ?$

$\sqrt{20} + \sqrt{45} = ?$

Exercise 5: rewrite these expression to the same surd

question	rewrite	Add or subtract of the same surd	answer
√32 + √18	√32=√2x16 =√2x4x4 =4√2 √18=√2x3x3 =3√2	= 4√2 +3√2	= 7√2
√3 + √12	√3 = √3 or 1√3 √12 = √3 x 2 x 2 = 2√3	=1√3+2√3	=3√3
√45+√45	√45 =1√45 1√45	=1√45 + 1√45	= 2√45
√27 + √48	√27 = √48 =	=	=
√75 + √12	√75 = √12 =	=	=

Now Let's tackle different types of questions

Operations with surds worksheet

1. Simplify the following:

(a) $\sqrt{5}+3\sqrt{5}$ **(b)** $3\sqrt{7}+2\sqrt{7}$ **(c)** $4\sqrt{5}-3\sqrt{6}+2\sqrt{5}-5\sqrt{6}$

(d) $-4\sqrt{2}+3\sqrt{3}-2\sqrt{3}-5\sqrt{2}$ **(e)** $3\sqrt{15}+2\sqrt{5}-2\sqrt{3}+6\sqrt{5}$ **(f)** $-7\sqrt{7}+3\sqrt{21}-2\sqrt{7}+\sqrt{21}$

2. Simplify the following:

(a) $\sqrt{8}+3\sqrt{2}$ **(b)** $\sqrt{12}-4\sqrt{3}$ **(c)** $4\sqrt{18}+3\sqrt{6}-\sqrt{2}$

(d) $-3\sqrt{48}+5\sqrt{27}$ **(e)** $2\sqrt{75}-5\sqrt{48}$ **(f)** $3\sqrt{8}+\sqrt{2}-\sqrt{98}$

3. Simplify the following:

(a) $\sqrt{7}\times\sqrt{5}$ **(b)** $\sqrt{15}\times\sqrt{2}$ **(c)** $\sqrt{6}\times\sqrt{6}$

(d) $\sqrt{21}\times\sqrt{3}$ **(e)** $\sqrt{6}\times\sqrt{10}$ **(f)** $2\sqrt{5}\times3\sqrt{11}$

(g) $4\sqrt{6}\times2\sqrt{5}$ **(h)** $2\sqrt{7}\times(-3\sqrt{14})$ **(i)** $6\sqrt{3}\times2\sqrt{15}$