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Strongly referencing Allen Forte's The Structure of Atonal Music: The Purpose and Applications of Pitch-Class Set Theory

The paper is going to elaborate on the set theorist's stance in the debate on atonal music.

Pitch-Class Set Theory: Purpose, Applications, and Core Operations

Introduction

In the realm of classical tonal music, the characteristics of the harmony and the melody were basically based on the tonal centers, functional harmony, and the movement across the keys. But it is over the course of the first century of the twentieth century that the works of several prominent composers like Arnold Schoenberg, Anton Webern, and Alban Berg, all of whom alienated themselves from tonality, came along. Their compositions utilized all twelve notes of the chromatic scale without any conductor, the tonal center. To make it easier to understand this new music, American theorist Allen Forte created a system called pitch-class set theory, which he described in his acclaimed book The Structure of Atonal Music (1973).

Forte's approach opens a new door for analysts and composers to describe, compare and understand what is referred to as atonal music. Rather than putting its emphasis on harmonic function, it investigates the building and interrelations of groups of pitches (which are termed pitch-class sets). This paper describes the objectives and uses of the pitch-class set theory and thoroughly explains its core operations with examples: set segmentation, normal order, prime form, rotation of notes in a set, intervallic inversion, interval vectors, and set complementation.

Forte's main objective was to establish a standard and unbiased language for the portrayal of atonal music. However, the situation for tonal music is different. There, the chords relate to one another through the functional harmony and relationships of keys. Atonal music, on the other hand, is coherent only by the content and structure of the intervals. So, Forte developed a mathematical apparatus for the characterizing pitch classes and their interrelations.

The scheme works by assigning a number from 0 to 11 to each note name. The numbers correspond to the twelve pitch-classes of the chromatic scale (C = 0, C♯/D♭ = 1, D = 2, …, B = 11). Notes separated by an octave are considered to be of the same pitch-class (for instance, C_4 and C_5 both correspond to 0). A set is defined as any combination of these pitch-classes.

Pitch-class set theory enables the analyst to:

1. Recognize and contrast the equivalent pitch collections by a common feature of the register, transposition, or inversion.
2. Characterize the atonal works via the recurring set types.

3. Differentiate the sets according to their intervallic content.
4. Reveal the complementary relationships that split the twelve-note chromatic "aggregate."
5. Establish a unified terminology for the discussions concerning non-tonal music.

According to Forte (1973), “the analyst can then reduce any atonal passage to a small number of basic pitch-class relations” (p. 3). The intent is to uncover the underlying coherence in what might initially appear to be mere random atonal material.

Set Segmentation

Set segmentation refers to splitting a music piece into smaller groups of notes with the same pitch class (or segments) that still sound musical together. The analyst would find out which notes are to be considered together through segmentation as atonal music has no usual phrase structure. These formations can be in the form of melodic gestures, chords, or ideas of motives recurring.

To illustrate, let’s say the notes C–E–F–A–B ♭ –D are played close in time. Their pitch class numbers are {0, 4, 5, 9, 10, 2}. We might divide these into two trichords: {0, 4, 5} and {9, 10, 2}. Each trichord is then subjected to internal interval structure analysis separately.

Segmentation is not a mechanical process; it requires superior musical judgment taking into consideration rhythm, contour, and phrasing. Forte (1973) and Straus (2005) both point out that segmentation is the first interpretive decision made by the analyst as it sets the comparison of materials throughout the analysis.

Normal Order

Normal order is the most compact ascending arrangement of a pitch-class set. This step guarantees prior writing sets in the same manner before their comparison.

Normal order can be found:

1. First of all, all the pitch-classes should be listed (mod 12) and arranged in ascending order.
2. Next, all cyclic permutations (rotations) should be tried - just by moving the first note to the end.
3. Finally take the order that has the smallest total span (that is, the distance from the lowest to the highest note).

Consider an example, the set {0, 4, 7, 9} can be put into different forms, such as: [0, 4, 7, 9], [4, 7, 9, 0], [7, 9, 0, 4], [9, 0, 4, 7]. The span is most compact in [0, 4, 7, 9], consequently, this is the normal order.

Normal order is one of the ways to formally analyze the music and it does so by ignoring the differences in the register or the order of appearance in the score (Straus, 2005).

Prime Form

The prime form is the least, most compact pitch-class set representation, shifted accordingly so that it starts with 0. It is a single representation of all the transpositionally or inversionally similar versions of a set.

The following steps need to be performed to get the prime form:

1. Normal order is to be taken first.
2. Then, the first note is moved to the 0 position.
3. Next, the original set is inverted ($x \to 12 - x \bmod 12$), reordered and then transposed to 0.
4. Finally, both are compared; the more compact on the left is the prime form.

To illustrate, {0, 1, 4, 6, 8} has a normal form of [0, 1, 4, 6, 8]. Its inversion [0, 2, 5, 6, 10] is less compact, leading to the conclusion that the prime form is [0, 1, 4, 6, 8] (Forte set class 5–14).

The prime form is a tool to mark the set-class - a group of related sets through transposition and inversion. Every class has its “Forte number” which is unique (for instance, 3–11, 5–14).

Rotation of Notes in a Set

Rotation (or cyclic permutation) changes the order of a set by moving its members around the circle. The result of this process can be a different musical emphasis but still the same set.

For example, [0, 4, 7] (C–E–G) can be shifted to [4, 7, 0] or [7, 0, 4]. They are all the same set {0, 4, 7}, but the different starts of pitch create different contours for each of the rotations.

Rotation is not only the means for the most compact arrangement (normal order) but also shows how composers can alter a motive yet retain its identity. Forte (1973) applied rotation as a tool for in-depth analysis of motivic relationships in Schoenberg and Webern, just like the other composers.

Intervallic Inversion

Intervallic inversion is akin to reflecting a set across an axis. Every note x is transformed into $(12 - x) \bmod 12$. By inversion, we get a set that is akin to its mirror image, thus unmasking the properties of symmetry and equivalence.

To illustrate, {0, 2, 5} inverts to {0, 10, 7} which, when transposed such that 0 is the starting point, results in [0, 3, 5]. The two are considered to be inversionally equivalent.

Inversion along with transposition is one of the two principal equivalence operations in pitch-class set theory. It is stated that two sets are part of the same class if one can be produced from the other by T (transposition) or I (inversion).

Interval Vectors

An interval vector is a visual representation that indicates the frequency of occurrence of each of the interval classes in the given set. There are interval classes with a total of six:

ic1 = the minor second/major seventh (1 or 11 semitones)

ic2 = the major second/minor seventh (2 or 10 semitones)

ic3 = the minor third/major sixth (3 or 9 semitones)

ic4 = the major third/minor sixth (4 or 8 semitones)

ic5 = the perfect fourth/fifth (5 or 7 semitones)

ic6 = the tritone (6 semitones)

To determine the vector, enumerate all possible note pairs in the set, get their interval classes, and tally them up. The outcome is represented in the format . For instance, the note set [0, 1, 4, 6, 8] corresponds to the interval vector <1, 2, 1, 3, 2, 1>.

The interval vector serves as a distinctive marker revealing the internal sound character of the set. Sets that possess the same vector are referred to as Z-related sets (Forte, 1973).

Set Complementation

In the case of the twelve-tone system, the set's complement comprises every pitch-class outside the scope of that set. As a case in point, if a given composition is based on {0, 1, 4, 6}, then the complement would be {2, 3, 5, 7, 8, 9, 10, 11}. The composers usually split the chromatic aggregate into sets that are not only complementary but also equal in size. The analysts may observe how these sets, through their mutual interactions, either alternate or cover the whole twelve-tone system. Complementation is a key concept in the understanding of aggregate completion in both early atonal and twelve-tone music (Forte, 1973).

Example Application: Schoenberg’s Three Piano Pieces, Op. 11, No. 1

Arnold Schoenberg’s Three Piano Pieces, Op. 11, No. 1 (1909) is one of the most frequently mentioned compositions regarding the development of pitch-class set theory. Allen Forte (1973) brilliantly analyzed this piece in The Structure of Atonal Music (pp. 43-55) and it is also included as a practice example in Joseph Straus’s Introduction to Post-Tonal Theory (2005, pp. 62-70). The previously mentioned analyses are the reasons for its being a perfect case for pitch-class set theory in practice.

At the beginning of Op. 11 No. 1, the opening gesture (measures 1-3) is bringing the pitch-classes {E-F-G-G♯-B♭-B} or numerically {4, 5, 7, 8, 10, 11}. Forte (1973) divides this fragment into intersecting trichords—{E, F, G} and {G♯, B♭, B}—and each one is a 3-3 [0,1,3] set class. This division illustrates the analytical operation of set segmentation whereby a continuous line of pitches gets divided into smaller, structurally meaningful subsets.

Analysts after identifying these trichords then proceed to determine their prime form and normal order. The {E, F, G} trichord converts to {4, 5, 7}; when its elements are reshuffled to achieve the tightest spacing (normal order [4, 5, 7]), and placed at 0, the prime form is [0, 1, 3]. The second trichord {G♯, B♭, B} or {8, 10, 11} has the same reduction to the prime form [0, 1, 3]. Forte (1973, p. 46) mentions that the same set-classes establish the connection between the melodic gestures and the harmonic structures in the first section through a similar motif.

The relationships of inversion are introduced by Schoenberg as the piece goes on (measures 4–10). According to Straus (2005, p. 66), the B♭ (pitch-class 10) initiated passage is a mirror image of the previous E-centered gesture by inversion around pitch-class 5 (F). When the inversion operation $In(x)=n-x$ mod 12 is applied, inverted forms are generated that have the same intervallic content, characteristic of the composer's use of motives. Initially, inversion changes [0, 1, 3] to [0, 2, 3] (Forte 3-5) which finally results in a contrast that is not easily perceptible but at the same time, the structural coherence is maintained.

In addition to this, the interval vector is another tool used by pitch-class set analysis to describe the internal intervallic content of each set. For the trichord [0, 1, 3], the interval vector is ⟨1, 1, 1, 0, 0, 0⟩ which indicates that there is one occurrence each of the interval classes 1, 2, and 3 in the set. This succinctly captures the essence of the content rig

By segmentation, identifying normal order and prime form, rotating and inverting subsets, comparing interval vectors, and complementation, Forte’s and Straus’s analyses disclose how Schoenberg's Op. 11 No. 1 achieves unity of motives without tonal repetition. Therefore, pitch-class set theory is a formal method that helps understand coherence in early atonal music.

Forte (1973, pp. 48-49) notes that in Schoenberg’s music the latter part of the trichords is made up of larger hexachords. He refers to one such hexachord, containing the pitches E, F, G, A♭, B♭, B or the numbers 4, 5, 7, 8, 10, 11, as Forte set-class 6-Z17 with prime form [0, 1, 3, 4, 6, 7] and interval vector ⟨3, 2, 3, 2, 3, 1⟩. Its complement, the set of the six pitch-classes {C, C♯, D, E♭, F♯, A} or {0, 1, 2, 3, 6, 9}, is 6-Z43, a Z-related pair that has the same interval vector but different normal forms. This makes the relationship between the hexachords a clear example of set complementation, which is one of the operations that shows the hidden symmetries across the atonal texture.

The use of interrelated techniques like segmentation, normal order and prime form identification, rotation and inversion of subsets, interval-vector comparison, and complementation, has led to the conclusion by Forte and Straus that Schoenberg's Op. 11, No. 1, achieves motivic unity without the repetition of tonal areas. Consequently, pitch-class set theory emerges as a formal tool for the interpretation of coherence in early atonal music.

Reasons of why does the Pitch-Class Set Theory matters or important

The theory of pitch-class sets is still, perhaps, the most significant analytical method for the music of the 20th and 21st centuries. It helps the analysts reveal the structure, balancing, and unity of the pieces that do not mean any tonal center traditionally. Straus (2005) asserts that “set theory gives us the power to describe and compare pitch collections in an objective manner, thereby revealing relationships that would otherwise remain unnoticed” (p. 50).

Composers, indeed, gain from this insight, too. They can sort out their atonal material in a coherent and eloquent way by examining the relations of pitch-class sets through transposition, inversion, and complementation.

Limitations of the Method

A few scholars argue that Forte's theory, despite its rigorousness, is primarily concerned with pitch relationships and completely ignores rhythm, dynamics, and timbre. In Buchler's view (1998), musical meaning is often determined by the context in which it occurs rather than by purely mathematical relations. Nevertheless, the criticisms do not prevent pitch-class set theory from being an important tool in the study of the structure of atonal music and its impact on contemporary analysis and composition.

In conclusion, Allen Forte's The Structure of Atonal Music (1973) offered an innovative system for interpreting atonal works. His pitch-class set theory takes the place of the tonal hierarchy which dominated the earlier centuries by representing the pitch-classes with exact relationships.

Analysts, through the mastery of the main operations of the theory i.e. segmentation, normal order, prime form, rotation, inversion, interval vectors, and complementation, can reveal the composer's skill of creating coherence through pitch organization in such composers as Schoenberg and Webern.

Thus, pitch-class set theory is not only a means of analysis but also a compositional method that enables both music theorists and composers to delve into the hidden logic of atonality.

References

Buchler, M. (1998). Relative saturation of subsets and interval cycles as a means for determining set-class similarity. *Music Theory Spectrum, 20*(1), 1–33.

Forte, A. (1973). *The structure of atonal music.* Yale University Press.

Forte, A. (1988). Pitch-class set analysis today. *Perspectives of New Music, 26*(2), 90–120.

Perle, G. (1977). *Serial composition and atonality: An introduction to the music of Schoenberg, Berg, and Webern* (5th ed.). University of California Press.

Rahn, J. (1980). *Basic atonal theory.* Longman.

Roig-Francolí, M. A. (2008). *Understanding post-tonal music.* McGraw-Hill.

Schoenberg, A. (1909/1994). *Three piano pieces, Op. 11 [Score].* In A. Schoenberg, *Complete works for solo piano* (M. Steuermann, Ed.). Belwin-Mills.

Straus, J. N. (2005). *Introduction to post-tonal theory* (3rd ed.). Pearson Prentice Hall.

Webern, A. (1909/1970). *Five movements for string quartet, Op. 5 [Score].* Universal Edition. Music