FIRST PRINCIPLE DIFFERENTIATION AND RULES OF DIFFERENTIATION

The rate of change of a function at a point is called a derivative. The derivative of a function at a point gives:

• the rate of change of the function at the point
• the slope (gradient) of the tangent to the function at the point

Definition of a derivative

The derivative of a function $y = f(x)$ is defined as $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
This formula is provided on the ***information sheet*** *in the final exam.*

NOTE: The notation we use for the derivative of $y = f(x)$ is
$f'(x)$ or y' or $\frac{dy}{dx}$ or $D_x[f(x)]$.
When we find the derivative of a function, we say we **differentiate** the function.

Derivative from first principle or using the definition

To differentiate from first principles (definition) use the formula below:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Class Exercise

Use the definition or first principle differentiation to differentiate the following expressions:

1. $f(x) = 2x - 3$
2. $f(x) = -4x$
3. $f(x) = 5x^2 - 4x + 2$
4. $f(x) = 3x^2$
5. $f(x) = -\frac{2}{x}$
6. $f(x) = 2x^3$

RULES OF DIFFERENTIATION

Rules 1. If $f(x) = b$ then $f'(x) = 0$ where b is a constant 2. If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ 3. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ 4. $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$	**e.g. 3** If $h(x) = 12$, then $h'(x) = 0$ The derivative of a constant is always = 0. If $k(x) = x^5$, then $k'(x) = 5x^4$ If $f(x) = x^5 + x^4$, then $\frac{d}{dx}f(x) = 5x^4 + 4x^3$ If $f(x) = 3x^5$ then $\frac{d}{dx}f(x) = 3 \times \frac{d}{dx}f(x)\ (x^5) = 3 \times 5x^4 = 15x^4$

PAST EXAM PRACTISE QUESTIONS

2010 March Paper 1 Q 10

10.1 Differentiate f from first principles: $f(x) = \frac{1}{x}$ (4)

10.2 Use the rules of differentiation to determine $\frac{dy}{dx}$ $if\ y = (2-5x)^2$ (3)

2015 Gauteng Prelim Paper 1 Q 10

10.1 Given $f(x) = -2x^2 + 1$

10.1.1 Show that the average gradient of the graph of f between the points where $x = 3$ and $x = 3 + h$ h, (h≠ 0) is $-12 - 2h$ (4)

10.1.2Use your answer in QUESTION 10.1.1 to calculate $f'(3)$ from first principles(2)

10.1.3 Determine the numerical value of the graph of f at $x = 0$ (1)

10.2 Differentiate with respect to x.Leave your answer with positive exponents.

10.2.1 $y = (4-2x)^2$ (3)

10.2.2 $y = \sqrt{x}\left(\sqrt{x} - a\right) + a$ (4)

2015 Western Cape Prelim Paper 1 Q8

8.1 If $f(x) = -2x^2$, determine $f'(x)$ from first principles (5)

8.2 Determine:

8.2.1

$$\frac{dy}{dx} \text{ if } y = \frac{2x^2 - 1}{\sqrt{x}}$$

(3)

8.2.2 $D_x [(3x-2)^2]$ (3)

8.3 Given: $y = \frac{1}{x^2}$.

Prove that the gradient of the curve is negative at each point on the curve where $x > 0$ (3)

2008 Higher Grade June Paper 1 Q6.1 and 6.2

6.1 Given: $f(x) = 3x^2 + 1$

6.1.1 Prove, by using **first principles**, that $f'(x) = 6x$. (6)

6.1.2 Hence, calculate:

(a) The gradient of the curve of f where $x = 2$ (2)

(b) The equation of the tangent to f at the point (2 ; 13) (3)

6.2 Determine $\frac{dy}{dx}$ if:

6.2.1 $y = 2x^2 + \sqrt{x^3}$ (3)

6.2.2 $2xy = 2x^2 - 7x + 6$ (4)

2014 March Paper 1 Q 10.1-10.3

10.1 Given: $f(x) = -\frac{2}{x}$

10.1.1 Determine $f'(x)$ from first principles. (5)

10.1.2 For which value(s) of x will $f'(x) > 0$? Justify your answer. (2)

10.2 Evaluate $\frac{dy}{dx}$ if $y = \frac{1}{4}x^2 - 2x$. (2)

10.3 Given: $y = 4\left(\sqrt[3]{x^2}\right)$ and $x = w^{-3}$

Determine $\frac{dy}{dw}$. (4)

2008 November Paper 1 Q8.1 and 8.2

8.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2$ (5)

8.2 Determine, using the rules of differentiation:

$$\frac{dy}{dx} \text{ if } y = \frac{\sqrt{x}}{2} - \frac{1}{6x^3}$$

(3)

2009 Higher Grade June Paper 1 Q6

6.1 Given: $f(x) = 7x + x^2$

6.1.1 Determine:
$\lim\limits_{x \to 0} \frac{f(x)}{x}$ (2)

6.1.2 Prove from **first principles** that $f'(x) = 2x + 7$. (5)

6.1.3 Determine the equation of the tangent to the graph of f at the point where $f'(x) = 1$. (5)

6.2 Given $y = kx + k$, where k is a constant.
Show that $\frac{dy}{dx} = \frac{y}{x+1}$. (3)

6.3 Determine $\frac{dy}{dx}$ in each of the following:

6.3.1 $y = x(x - x^{-1})^2$ (4)

6.3.2 $\sqrt{x}.y = 2x + 1$ (4)

2013 Higher Grade June Paper 1 Q6

6.1 Given: $f(x) = 1 - x^2$

Determine the derivative, $f'(x)$, from **first principles**. (5)

6.2 Differentiate the following with respect to x:

6.2.1 $y = \frac{x^2}{2} - \frac{3}{x^3}$ (3)

6.2.2 $g(x) = \left(x + \frac{3}{\sqrt[3]{x}}\right)\left(x - \frac{3}{\sqrt[3]{x}}\right)$ (4)

6.3 The straight line having equation $y = 4x + c$ is a tangent to the curve having equation $h(x) = 2x^3 + 7x^2 + 4x - 4$. Calculate the value of c if $x < 0$. (8)

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2011 Higher Grade June Paper 1 Q6

6.1 Determine the derivative of $f(x) = x^2 - x$ from **first principles**. (5)

6.2 Determine $h'(x)$:

6.2.1 $h(x) = \pi x - \frac{4}{x}$ (3)

6.2.2 $h(x) = \dfrac{x^3 + \sqrt{x^3}}{x}$ (4)

6.3 The equation of a tangent at the point $(-1\ ;\ 3)$ on the curve $f(x) = ax^3 + bx$ is $y = x + 4$. Calculate the values of a and b. (6)

2016 March Paper 1 Q8.1-8.2

8.1 Determine $f'(x)$ from first principles if $f(x) = -x^2 + 4$. (5)

8.2 Determine the derivative of:

8.2.1 $y = 3x^2 + 10x$ (2)

8.2.2 $f(x) = \left(x - \dfrac{3}{x}\right)^2$ (3)

2013 Gauteng June Paper 1 Q8.1-8.2

8.1 If $f(x) = 2 - x^2$, determine $f'(x)$ from first principles. (5)

8.2 Determine $\dfrac{dy}{dx}$ if:

8.2.1 $y = (x^2 - 1)(x^2 + 1)$ (2)

8.2.2 $y - 2\sqrt{x} = \dfrac{1}{2x^3}$ (3)

2012 Higher Grade June Paper 1 Q6

6.1 Given: $f(x) = ax^2 - 3x$

6.1.1 Determine the derivative, $f'(x)$, from **first principles**. (5)

6.1.2 Calculate the values of a if $f'(a) = 50$. (3)

6.2 Differentiate the following with respect to x:

6.2.1 $h(x) = \left(x + \frac{1}{x}\right)^2$ (4)

6.2.2 $y = \frac{\sqrt{x} + 3}{3\sqrt{x}}$ (3)

6.3 The line $y = 4x - 3$ is a tangent to the curve $g(x) = 2ax^3 - bx^2$ at $x = 1$. Determine the values of a and b. (8)

2015 Free State Prelim Paper 1 Q7.1-7.2

7.1 Differentiate, using the first Principles, the function $f(x) = -\frac{3}{x}$ (5)

7.2 Differentiate:

7.2.1 $y = \frac{x^2 - 3x + 1}{x^3}$ (4)

7.2.2 $y = \frac{\sqrt{t}}{2} - \frac{4}{7t^5}$ (4)

2010 November Paper 1 Q 8

8.1 Differentiate $g(x) = x^2 - 5$ from first principles. (5)

8.2 Evaluate $\frac{dy}{dx}$ if $y = \frac{x^6}{2} + 4\sqrt{x}$. (3)

8.3 A function $g(x) = ax^2 + \frac{b}{x}$ has a minimum value at $x = 4$. The function value at $x = 4$ is 96. Calculate the values of a and b. (6)

2008 Standard Grade June Paper 1 Q6

6.1 Given: $f(x) = -3x^2$

6.1.1 Complete and simplify the following: $f(x+h) = \ldots$ (2)

6.1.2 Hence determine $f'(x)$ from **first principles**. (4)

6.1.3 Write down the gradient of the tangent to the graph of f at $x = -2$. (2)

6.1.4 Determine the equation of the tangent to the graph of f at $x = -2$. (4)

6.2 Differentiate with respect to *x*:

6.2.1 $y = x(x^2 - 3)$ (3)

6.2.2 $y = \frac{3}{x} + x^{\frac{1}{3}}$ (3)

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2009 Standard Grade June Paper 1 Q6

6.1 Determine $f'(x)$ from **first principles** if $f(x) = 4x + 1$. (5)

6.2 Given: $f(x) = x^3 + 1$

6.2.1 Determine the average gradient of f between the points where $x = 1$ and $x = 4$. (4)

6.2.2 Determine the gradient of the tangent to the graph of $y = x^3 + 1$ at the point where $x = -2$. (3)

6.3 Determine $\frac{dy}{dx}$ for each of the following:

6.3.1 $y = x^6 - 2x^3 + 6$ (3)

6.3.2 $y = 2\sqrt{x} - \frac{1}{x^3}$ (4)

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2010 Standard Grade June Paper 1 Q7.1-7.2

7.1 Given: $f(x) = 4x - 3$

Use the definition (first principles) to find $f'(x)$, the first derivative of $f(x)$. (5)

7.2 Differentiate each of the following with respect to x:

7.2.1 $g(x) = x^4 + x^{-4}$ (2)

7.2.2 $h(x) = \frac{x+2}{\sqrt{x}}$ (4)

2011 Standard Grade June Paper 1 Q5

5.1 Given: $f(x) = \frac{1}{2}x^2$

Use the definition (first principles) to find $f'(x)$, the first derivative of $f(x)$. (5)

5.2 Determine $\frac{dy}{dx}$ if:

5.2.1 $y = 4x^{\frac{3}{4}} + 3x$ (2)

5.2.2 $y = \frac{6x^3 - 3}{3x}$ (3)

2014 Exemplar Paper 1 Q 8.1-8.2

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 2$. (5)

8.2 Determine $\frac{dy}{dx}$ if $y = 2x^{-4} - \frac{x}{5}$. (2)

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2009 March Paper 1 Q 11

11.1 Differentiate f by first principles where $f(x) = x^2 - 2x$. (5)

11.2 Evaluate:

11.2.1 $D_x[(x^3 - 3)^2]$ (3)

11.2.2 $\frac{dy}{dx}$ if $y = \frac{4}{\sqrt{x}} - \frac{x^3}{9}$ (3)

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