Engineering Science N4

Chapter 2 Relative Velocity

2.1 Introduction

The concept of relative velocity may be understood from the four examples of one-dimensional relative velocity as illustrated below.

One dimensional Relative Velocity is actually a plane or ship having a velocity in one direction only. This direction may be the North South axis or the West South axis as the case may be.

Two dimensional Relative Velocity is a body travelling in two directions. A ship or plane travelling in the North Western direction has a velocity component in two directions namely, Northern direction and Western direction.

The velocity of a wind blowingin the Northern direction will definitely increase the velocity of a plane flying in the same direction as the wind.

The velocity of a wind blowing in the Northern direction will definitely decrease the velocity of a plane flying in the Southern direction.

2.2 Formula and Terminology

	Table 1.1		
		Formula	Unit
1.	Distance	velocity X time	meters, m
2.	Velocity	Distance/time	Meters/second, m/s
3.	Time	Distance/velocity	seconds, s
4.	Horizonal Component	Vcos ϴ	m/s
5.	Vertical Component	Vsin ϴ	m/s
6.	Cosine Rule	R^2= $WY^2 + WX^2 - 2WY.WX.\cos\Theta$	
7.	Sine Rule		
8.	Pythagoras Theorem	$YX^2 = WY^2 + WX^2$	

Vector: Velocity and Force are vectors because they have magnitude and direction.

Vector Parallelogram: The first two vectors of a two dimensional relative velocity problem are the first part of the parallelogram. The corresponding opposite side of each of the vectors completes the parallelogram.Drawing the resultant is carried out by drawing a line from the original point of the first two vectors to the opposite corner of the parallelogram. (See Fig.) Vector Triangle: This triangle is half of the vector parallelogram. The vectors on this triangle are drawn thus:

a) tail of first vector to head of first vector.

b) tail of second vector to head of second vector.

c) the resultant is drawn from the tail of the first vector to the head of the second vector.

(See Fig.)

Potential Energy of a body is the energy a body possesses based on its vertical height above the ground.

2.3 One directional Relative Velocity

Example 1.

A plane is flying in the Northward direction at a velocity of 185km/h. A wind of 25km/h is blowing in the same Northward direction. Draw and label the vector diagram based on the details given. Calculate the velocity of the plane.

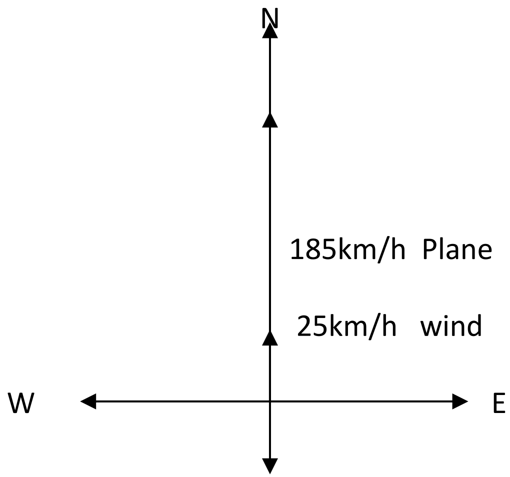


Fig. 2.1 One Dimensional Relative Velocity

The resultant velocity is going to increase because the wind is blowing in the same direction as the plane. The plane's velocity and the wind's velocity are both considered as positive.

Resultant = (185 + 25)km/h
=210km/h Northern direction

Example 2.
A plane is flying in the Northward direction at a velocity of 185km/h. A wind of 25km/h is blowing in the same Southward direction. Draw and label the vector diagram

based on the details given. Calculate the velocity of the plane.

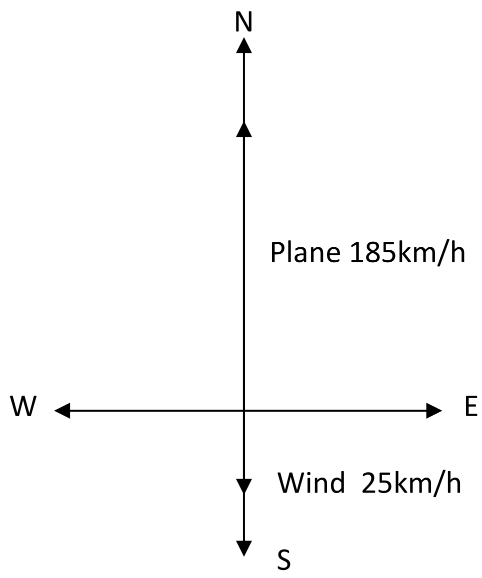


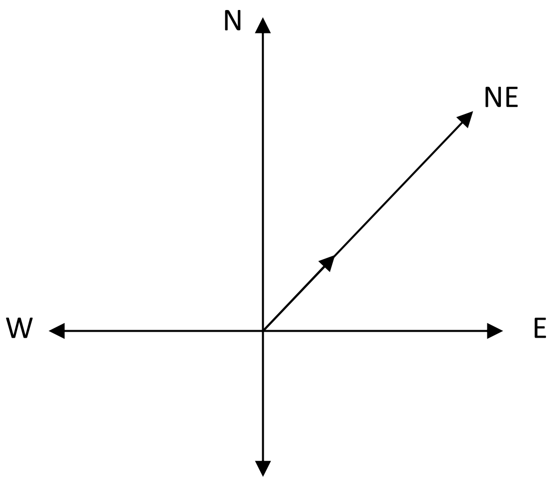
Fig. 2.2

The resultant velocity is going to decrease because the wind is blowing in the opposite direction to the plane. The plane's velocity is considered as positive and the wind's velocity is considered as negative.

Resultant = (185 - 25)km/h
= 160km/h Northern direction

Example 3.

A plane is flying in the North Eastern direction at a velocity of 225km/h. A wind of 65km/h is blowing in the same North Eastern direction. Draw and label the vector diagram based on the details given. Calculate the velocity of the plane.



Resultant = (225 + 65)km/h

=290km/h NE

Fig 2.3 One directional relative Velocity

Example 4.

A plane is flying in the North Eastern direction at a velocity of 225km/h. A wind of 65km/h is blowing in the same South Western direction. Draw and label the vector diagram based on the details given. Calculate the velocity of the plane.

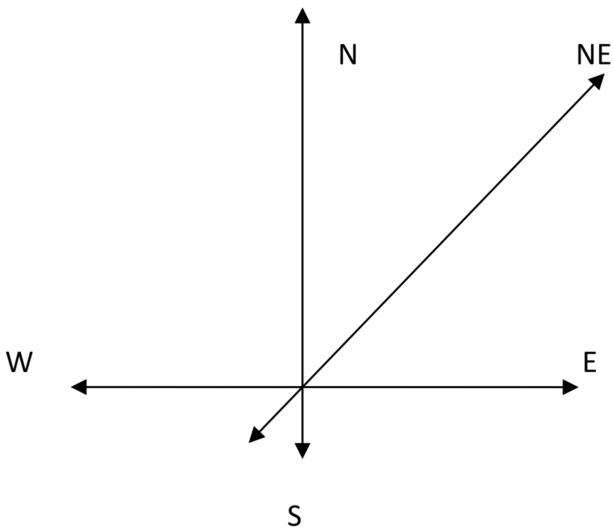


Fig. 2.4 One directional relative Velocity

Resultant = (225 - 65)km/h
=160km/h NE

2.3 Two directional Relative Velocity

This is the velocity of a body in two directions. The velocity of a plane in the North Eastern direction has both Northern component and Eastern component defined by Vsin Ɵ and Vcos Ɵ respectively.

2.3.1 Bearing and Direction

Example 1.
A plane is flying in the Eastward direction at a velocity of 125km/h. A wind of 45km/h is blowing in the same Northward direction. Draw and label the vector diagram based on the details given. Calculate the velocity of the plane.

Resultant = $[125km/h)^2 + (45km/h)]^{0.5}$
= 132.8533 km/h
tan Ɵ= [45/125]
Ɵ = $\tan^{-1}$ [45/125] = 19.799 degrees 132.8533 km/hE
19.799^o N

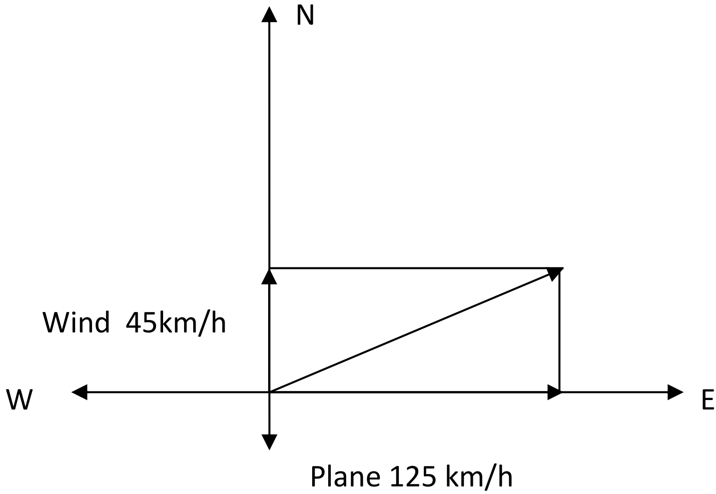


Fig. 2.5 Vector Parallelogram

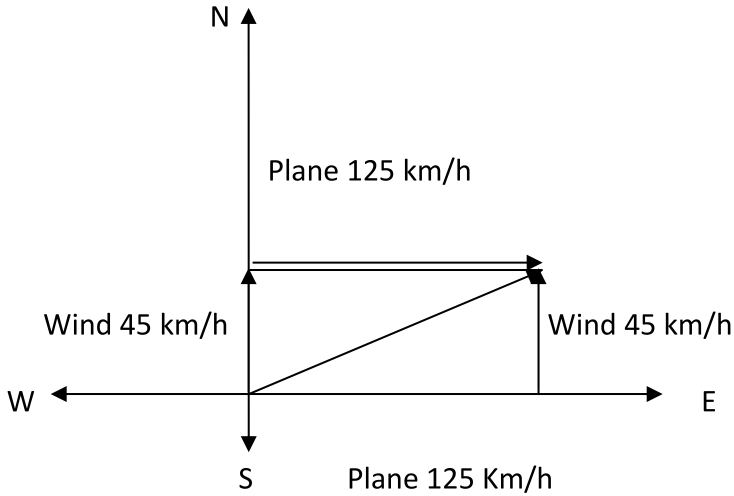


Fig. 2.6 Vector Parallelogram and Vector Triangle.

Resultant = $[125km/h)^2 + (45km/h)]^{0.5}$
= 132.8533 km/h
tan Ө= [45/125]
Ө = $\tan^{-1}$ [45/125] = 19.799 degrees 132.8533 km/hE
19.799^o N

Example 2.

A plane is flying in the Eastward direction at a velocity of 155km/h. A wind of 85km/h is blowing in the North eastern direction. Calculate the velocity of the plane. Draw and label the vector diagram based on the details given. Also label the resultant in terms of magnitude and direction.

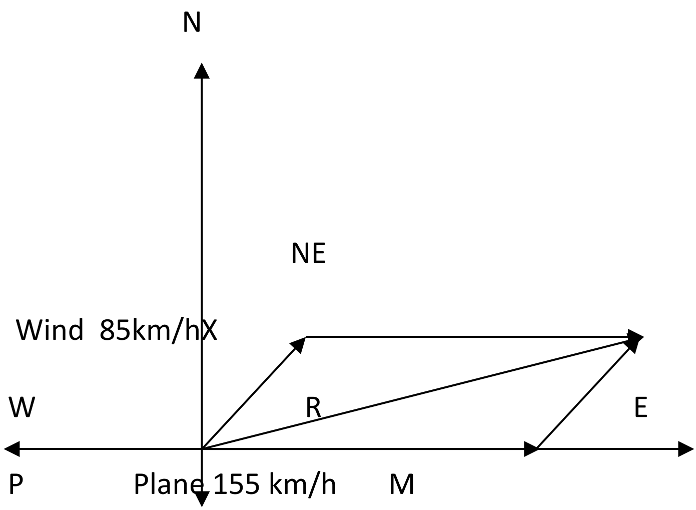


Fig. 2.7 Resultant of plane flying at 155 km/h and wind blowing at 85 km/h.

Apply the Cosine Rule

$R^2 = PM^2 + MX^2 - 2(PM)(MX)\cos \Theta$
Θ = Angle PMX

The Cosine Rule is applied to solve a triangle when the two sides that form a specific and are known. The angled formed is also known and the letter R or Resultant is the side opposite the known angle.
(Refer to Fig. *** above).
$R^2 = (155)^2 + (85)^2 - 2(155)(85)\cos(180-45)$
R= 223.3434 km/h
Apply the Sine Rule to solve for the angle between R and the Western and Eastern axis.

$$\frac{\sin 135}{223.3434\text{km/h}} = \frac{\sin(\text{Angle MPX})}{85\text{km/h}}$$

$$\text{Sin(Angle MPX)} = \frac{85\text{km/h X} \sin 135}{223.3434\text{km/h}}$$

Angle MPX = $\sin^{-1}(0.26911)$
$=15.6113^0$

The velocity of the plane has increased from 155km/h to 223.3434km/h due to the velocity of the wind. In addition to this, the original angle of flight of the plane has been altered to N 15.6113^0 E.
R= 223.3434 km/h N 15.6113^0 E.

Example 3.
A plane is flying in the Northward direction at a velocity

of 195km/h. A wind of 25km/h is blowing in the Northeastern direction.
Calculate the velocity of the plane.
Draw and label the vector diagram based on the details given.
Also label the direction and velocity that the pilot has to fly in order for the plane to arrive at the required destination.

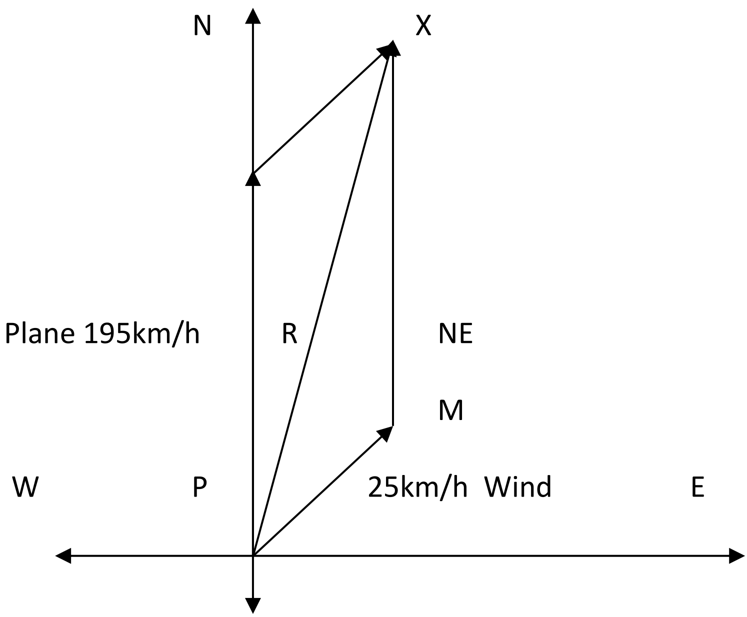


Fig 2.8 Two directional relative velocity

$R^2 = (195)^2 + (25)^2 - 2(195)(25)\cos(180-45)$
R= 213.411 km/h

$$\frac{\sin 135}{213.411\text{km/h}} = \frac{\sin(\text{Angle MPX})}{195\text{km/h}}$$

$$\text{Sin(Angle MPX)} = \frac{195\text{km/h X} \sin 135}{213.411\text{km/h}}$$

Sin(Angle MPX) = 0.6461

Angle MPX = $\sin^{-1}$ (0.6461)
= 40.2481^0

$\Theta = 40.2481^0 + 45^0 = 85.2482^0$
R= 213.411 km/h E 85.2482^0 N

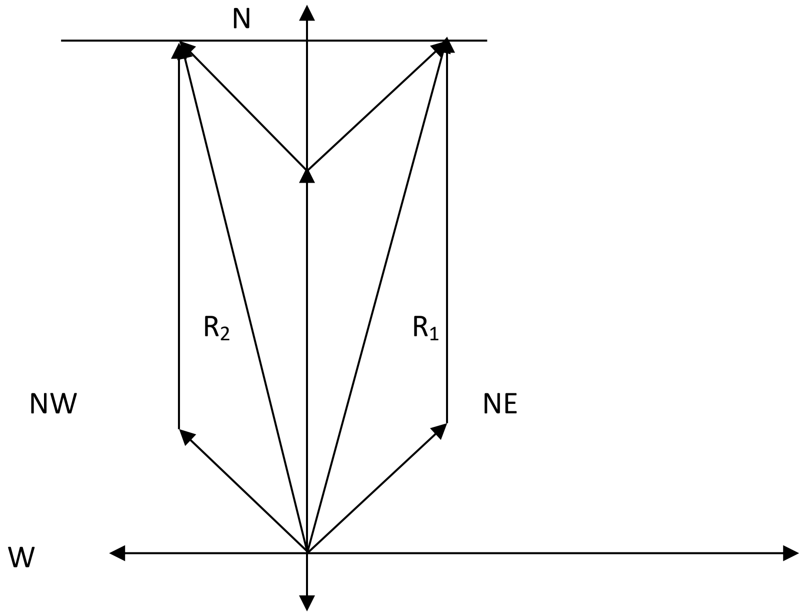


Fig. 2.9 New Vector in terms of velocity and direction

Usually, when the wind has a component of velocity in the same direction as the plane, the velocity of the plane increases.
The next example confirms this concept.

Example 4.
A plane is flying in the Northward direction at a velocity of 255km/h. A wind of 75km/h is blowing in the

Southwestern direction. Calculate the velocity of the plane in terms of velocity and direction. Draw and label the vector diagram based on the details in the problem and your final calculations. Explain the increase or decrease of the velocity of the plane. How does the wind change the direction of the plane.

$R^2 = (255)^2 + (75)^2 - 2(255)(75)\cos(45)$
R= 208.8137 km/h

$$\frac{\sin 45}{208.8137\text{km/h}} = \frac{\sin(\text{Angle MPX})}{255\text{km/h}}$$

Sin(Angle MPX) = 0.8635

Angle MPX = $\sin^{-1}(0.8635)$
$= 59.7119^0$

R= 208.8137 km/h W59.7119^0 N

It would be very important to note that the point or corner labeled M is the angle that is made by the two vectors of the plane and the wind.The point P is at the origin of the graph and it is the tail of the resultant vector.

N

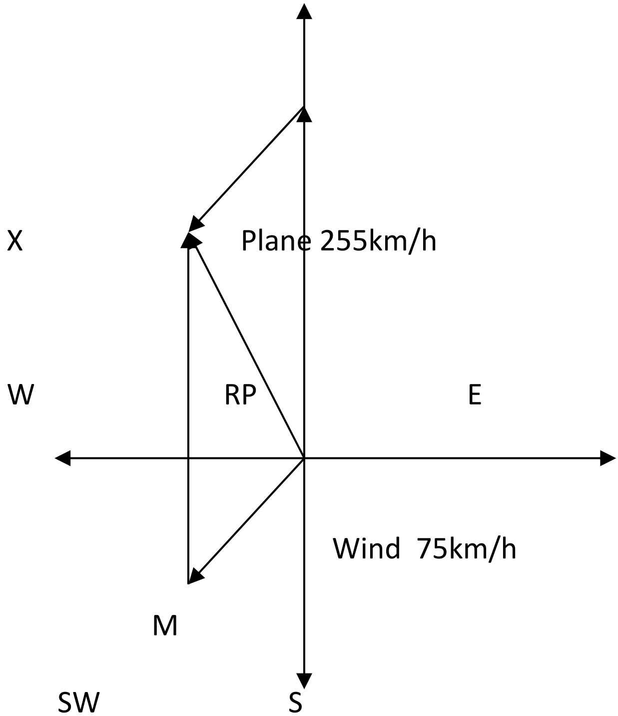


Fig 2.10 Two directional relative velocity

2.3.2 Resolving Vectors into horizontal and vertical components

The second method of calculating the magnitude and direction of a resultant vector is based on resolving the vectors into two components namely:

a) Horizontal Components
b) Vertical Components

The sum of the vertical and horizontal components for both plane and wind as the case may be is therefore used to draw the corresponding vector rectangle of the resultant.

Example 1.
A plane is flying in the Eastward direction at a velocity of 155km/h. A wind of 85km/h is blowing in the North eastern direction. Calculate the velocity of the plane. Draw and label the vector diagram based on the details given. Also label the resultant in terms of the corresponding vector rectangle.

Table 1.2		
	Vertical Component	Horizontal Components
Plane	0 km/h	+155km/h
Wind	(85sin45)km/h	(85cos45)km/h
Total	60.1041kim/h	215.1041km/h
Resultant	$(V_H+V_V)^{0.5}$= 223.3434km/h	
Direction	Tan^{-1}=(V_V/ V_H)= 15.6114^0	

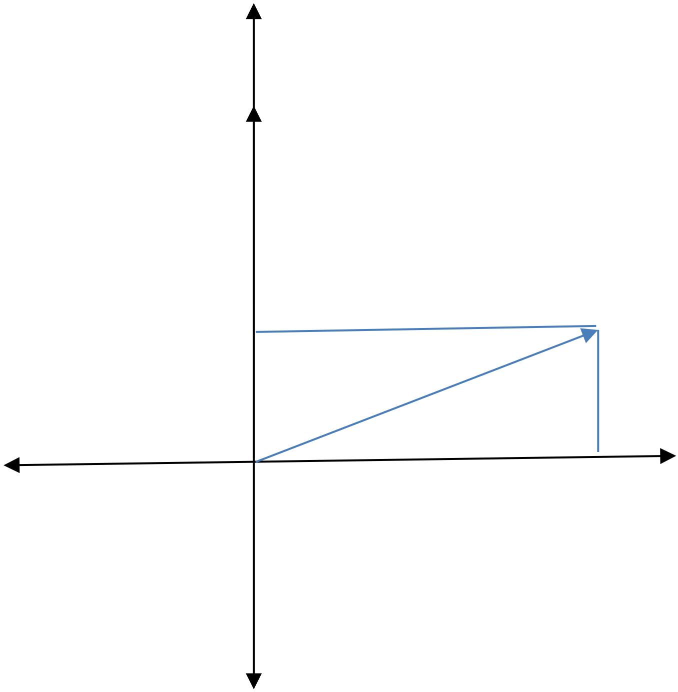


Fig. 2.11

Example 2.

A plane is flying in the Northward direction at a velocity of 195km/h. A wind of 25km/h is blowing in the Northeastern direction.

Calculate the velocity of the plane.

Draw and label the vector diagram based on the details given.

Table 1.3		
	Vertical Component	Horizontal Components
Plane	195km/h	0km/h
Wind	25 (sin45)km/h	25 (sin45)km/h
	212.6777km/h	17.6777km/h
Resultant	$(V_H+V_V)^{0.5}$= 213.4111km/h	
Direction	$\text{Tan}^{-1}(V_V/V_H)$= 85.2423^0	

Example 4.

A plane is flying in the Northward direction at a velocity of 255km/h. A wind of 75km/h is blowing in the Southwestern direction. Calculate the velocity of the plane in terms of velocity and direction. Draw and label the vector diagram based on the details in the problem and your final calculations. Explain the increase or decrease of the velocity of the plane. How does the wind change the direction of the plane.

Table 1.4		
	Vertical Component	Horizontal Components
Plane	255km/h	0 km/h
Wind	-75(sin45)km/h	-75(cos45)km/h
Total	201.9669km/h	-53.033km/h
Resultant	$(V_H+V_V)^{0.5}$= 208.8134km/h	
Direction	$Tan^{-1}(V_V/ V_H)$= 75.2872^0	

Example 5

A boat is travelling in the Northward direction at a velocity of 255km/h. A wind of 75km/h is blowing in the Northeastern direction. Calculate the velocity of the plane in terms of velocity and direction. Draw and label the vector diagram based on the details in the problem and your final calculations. Explain the increase or decrease of the velocity of the plane. How does the wind change the direction of the plane.

Table 1.5		
	Vertical Component	Horizontal Components
Plane	255km/h	0 km/h
Wind	75(sin45)km/h	75(cos45)km/h
Total	308.033km/h	53.033km/h
Resultant	$(V_H+V_V)^{0.5}$= 312.5649km/h	
Direction	$Tan^{-1}(V_V/ V_H)$= 80.2313^0	

Example 6

The resultant velocity of a boat travelling in a river is 312.5649km/h E 80.2313^0 N. The river current is flowing in the Northeastern direction at a velocity of 75km/h. Calculate the initial vel;ocity of the boat in terms of magnitude and direction. Draw and label this problem in a vector parallelogram.

Table 1.6		
	Vertical Component	Horizontal Components
Plane		
Wind	75(sin45)km/h	75(cos45)km/h
Total	308.033km/h	53.033km/h
Resultant	312.5649km/h	E 80.2313^0 N
	312.5649(sin80.2313^0)km/h	312.5649(cos80.2313^0)km/h

The word Relative in the term Relative velocity may be explained from the point of perspective of the above example in which the resultant velocity and the velocity of the wind are given and we are required to calculate the velocity of the plane.

This example was restructured based on example 5 above.

Therefore,
$312.5649(\sin 80.2313^0)$km/h - $75(\sin 45)$km/h=

And

$312.5649(\cos 80.2313^0)$km/h - $75(\cos 45)$km/h=