Utility Function

Suppose that you prefer $x = \{x_1, x_2\}$ over $y = \{y_1, y_2\}$. That is, $x \succ y$. Then a utility function is a function such that:

$$U(x) > U(y)$$

Monotonic Transformation

Monotonic transformations are transformations in the utility function that don't alter the order of the individual's bundle preferences.

Suppose the following utility function:

$$U(x_1, x_2) = x_1 x_2$$

Now the following transformation is done:

$$f[U(x_1, x_2)] = \ln[U(x_1, x_2)] = \ln \overbrace{(x_1 x_2)}^{U(x_1, x_2)}$$

$$= \ln x_1 + \ln x_2$$

This is a monotonic transformation, because for every two pairs of bundles, x' and x'', such that $U(x') > U(x'')$ we have:

$$f[U(x')] > f[U(x'')]$$

Conclusion: $U(x_1, x_2) = x_1 x_2$ and $\overbrace{V(x_1, x_2)}^{f[U(x_1, x_2)]} = \ln x_1 + \ln x_2$ represent the same preferences.

Common monotonic transformations:

$$\pm\, constant;\ /\ or\ \times\ constant;\ ln;\ e$$

- $e = 2{,}71 \ldots$

Deriving Indifference Curves from Utility

To derive indifference curves from utility functions, we do the following procedure:

1. Let $u(x_1, x_2)$ be called k, that is, $u(x_1, x_2) \equiv k$.
2. Isolate x_2, making x_2 as function of x_1 and k.
3. For every value of k, there is a different indifference curve.

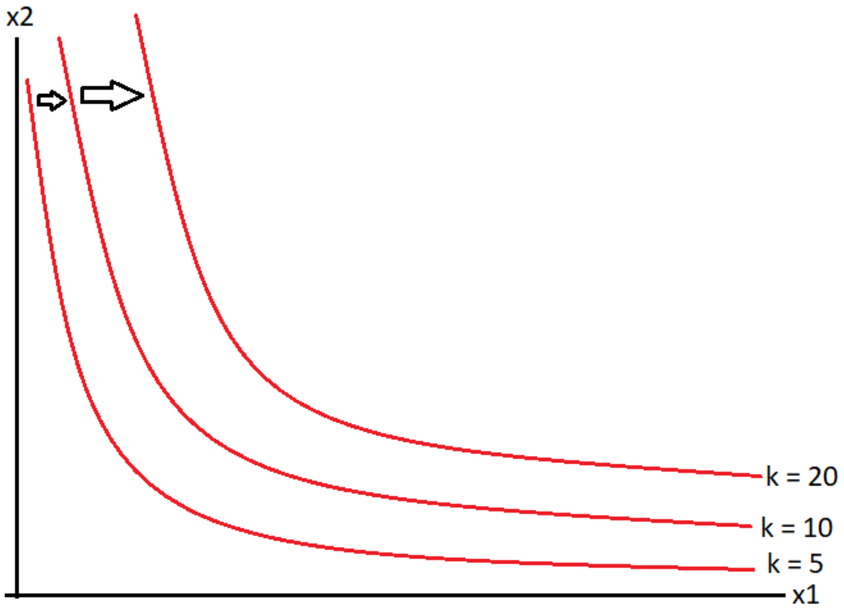
Examples:

$$\overbrace{u(x_1, x_2)}^{k} = x_1 x_2$$

$$\boldsymbol{x_2 = \frac{k}{x_1}}$$

$$\overbrace{u(x_1, x_2)}^{k} = x_1^{\alpha} x_2^{\beta}$$

$$\boldsymbol{x_2 = \left(\frac{k}{x_1^{\alpha}}\right)^{\frac{1}{\beta}}}$$



Perfect Complements

$$\overbrace{u(x_1, x_2)}^{k} = \min\{ax_1, bx_2\}$$

1º) $ax_1 > bx_2$:

$$k = bx_2$$

$$x_2 = \frac{k}{b}$$

2º) $ax_1 < bx_2$:

$$k = ax_1$$

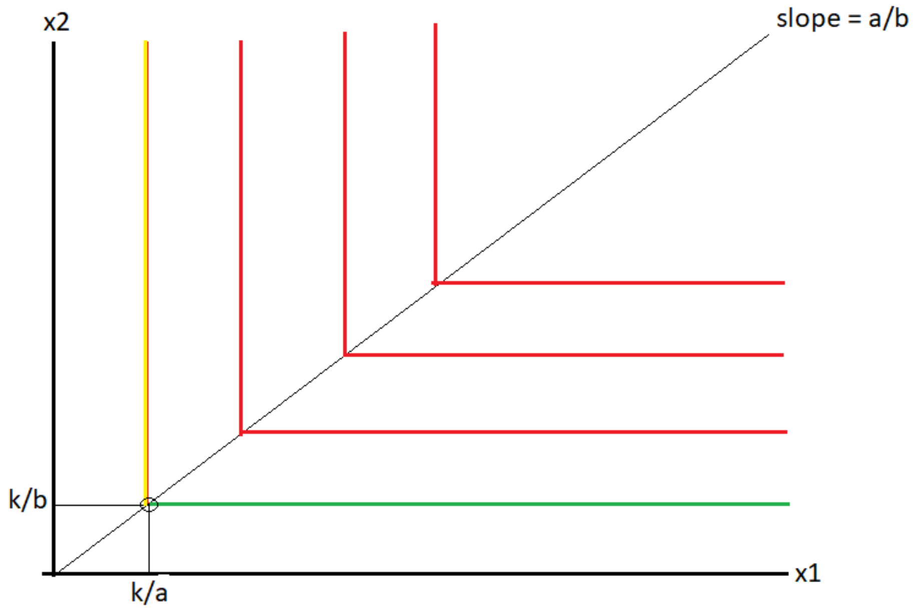
$$x_1 = \frac{k}{a}$$

3º) $ax_1 = bx_2$:

$$k = ax_1 = bx_2$$

$$ax_1 = bx_2$$

$$x_2 = \left(\frac{a}{b}\right) x_1$$

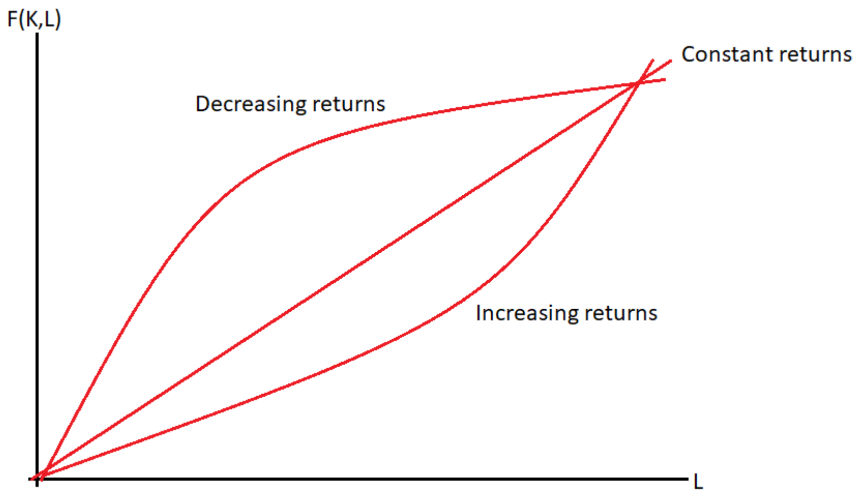


Cobb-Douglas Utility Functions

$$u(x_1, x_2) = x_1^c x_2^d$$

$$F(L, K) = L^c K^d$$

- $c + d = 1$: **Constant returns to scale:** if the firm doubles the inputs (L and K), the production increases by a factor of 2.
- $c + d < 1$: **Decreasing returns to scale:** if the firm doubles the inputs (L and K), the production increases by a factor less than 2.
- $c + d > 1$: **Increasing returns to scale**: if the firm doubles the inputs (L and K), the production increases by a factor more than 2.



Deriving the formula of MRS from the total derivative

Suppose we apply a **total derivative** to $u(x_1, x_2)$, then we have:

$$dU = \overbrace{\frac{\partial U}{\partial x_1}}^{MU_1} dx_1 + \overbrace{\frac{\partial U}{\partial x_2}}^{MU_2} dx_2$$

The MRS is calculated on an indifference curve, so we have to have $dU = 0$. Then:

$$0 = MU_1 dx_1 + MU_2 dx_2$$

$$MU_2 dx_2 = -MU_1 dx_1$$

$$\boldsymbol{MRS = \frac{dx_2}{dx_1} = -\frac{MU_1}{MU_2} = -\frac{\left(\frac{\partial U}{\partial x_1}\right)}{\left(\frac{\partial U}{\partial x_2}\right)}}$$

MRS:

- Graphical interpretation: the slope of the indifference at a certain point.
- Economic interpretation: how many units of x_2 are you willing to give up in order to obtain 1 unit of x_1.