Shear stress in "Laminar" Flow

7.1 Newton's Viscosity relation

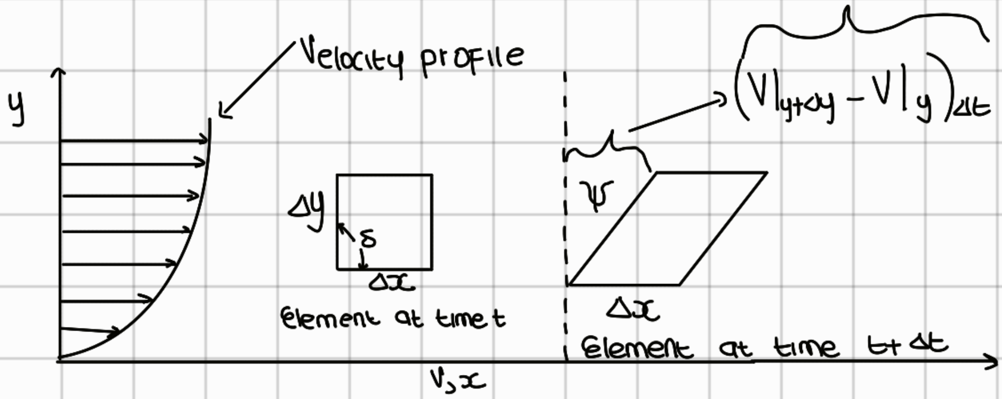
$$\text{Shear Modulus} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$\mu(\text{viscosity}) = \frac{\text{Shear stress}\,(\tau)}{\text{Rate of Shear strain}}$$

μ - Tendency of a fluid to resist deformation when acted upon by shear stress

$$\mu = \lambda(T, \overset{\text{composition}}{x}, P)$$

$$\mu \neq \lambda(\text{rate of shear strain})$$



$$\tan\psi = \frac{\left(V|_{y+\Delta y} - V|_y\right)\Delta t}{\Delta y} = \frac{\Delta d}{\Delta y}$$

$$\psi = \arctan\left(\frac{\left(V|_{y+\Delta y} - V|_y\right)\Delta t}{\Delta y}\right)$$

$$\psi = \frac{\tau}{2} - \delta\Big|_{t+\Delta t} \quad \ldots \quad (3)$$

Since : $\dfrac{d\delta}{dt} = \lim\limits_{\Delta t \to 0} \dfrac{\delta|_{t+\Delta t} - \delta|_t}{\Delta t}$

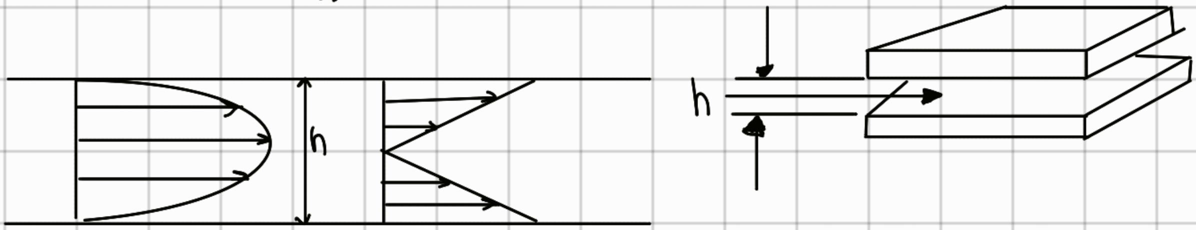
$$-\frac{d\delta}{dt} = -\lim_{\substack{\Delta t \to 0 \\ (\Delta x, \Delta y \to 0)}} \left[\frac{\frac{\tau}{2} - \left[\frac{\left(V|_{y+\Delta y} - V|_y\right)\Delta t}{\Delta y}\right] - \frac{\pi}{2}}{\Delta t}\right]$$

$$= \left[\lim_{\Delta x, \Delta y \to 0} \frac{(\;)_{y+\Delta y} - (\;)_{y}}{\Delta y} \right] = \frac{dv}{dy}$$

$$-\frac{d\delta}{dt} \equiv \text{rate of shear strain at a point}$$

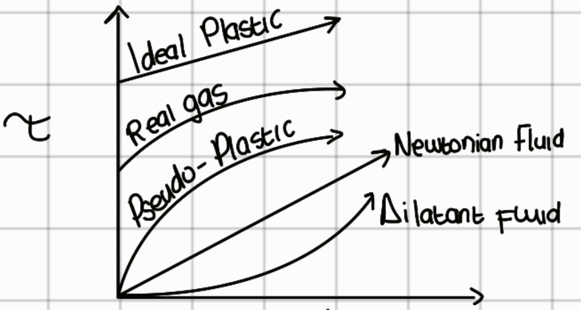
$$\mu = \frac{\tau}{(du/dy)}$$

$$\tau = \mu \left(\frac{dv}{dy}\right)$$



7.2 Non-Newtonian Fluids

Does not obey Newton's law of viscosity



$\frac{dv}{dy}$ (Rate of shear strain)

$$\tau = \mu \left(\frac{dv}{dy}\right)$$

In '∀' fluids, shear stress depends upon the rate of shear strain, albeit differently

7.3 Viscosity

For gases $\mu \propto T^{\beta}$

$$\beta \geq \frac{1}{2}$$

$$\mu = \frac{\tau}{(dv/dt)} = \frac{F/L^2}{(L/t)/L} = \frac{Ft}{L^2}$$

F = force

t = time

L = length

$$F = ML/t^2$$

$$\mu = \frac{ML}{t^2}\frac{t}{L^2} = \frac{M}{Lt}$$

$$v = \frac{\mu}{\rho} = \frac{M}{Lt}\frac{L^3}{M}$$

$$v = L^2/t$$

7.4 Shear stress in Multidimentional Laminar flows of a newtonian fluid

$$\text{Viscosity} = \frac{\text{Shear stress}}{\text{rate of shear strain}}$$