Physics Theory

Simple Harmonic Motion:

Periodic motion is a type of motion that causes the object to regularly return to a given position after a time interval. Simple harmonic motion is the motion when the force is directed towards equilibrium position, and is proportional to the position of the object relative to some equilibrium.

The motion of a spring-mass system with no frictional force can be modelled as SHM.
According to Hooke's Law:

$$\vec{F}_s = -k\vec{x}$$

As we can see the force is always opposite of the displacement from equilibrium.
Knowing that:

$$\vec{F}_s = -k\vec{x} = m\vec{a}$$

We can conclude that

$$-k\vec{x} = m\frac{\mathrm{d}^2\vec{x}}{\mathrm{d}t^2}$$

$$\vec{a} = \frac{\mathrm{d}^2\vec{x}}{\mathrm{d}t^2} = -\frac{k}{m}\vec{x}$$

As we can see, the acceleration is proportional to the displacement, and is not constant.
Therefore the kinematics equations are not applicable. The solution to this differential equation is

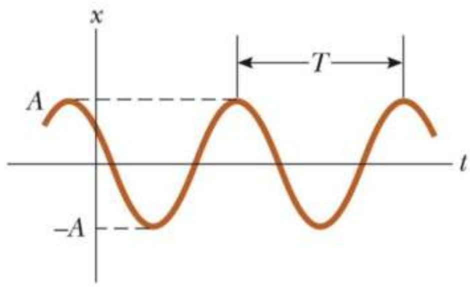
$$x(t) = Acos(\omega t + \phi)$$

Where A is the amplitude, phi is the phase constant and omega the angular frequency.
If the block is released from position x = A, then

$$\vec{a} = -\frac{kA}{m}$$

The block will continue to oscillate between +A and -A, these positions are the turning points of motion.

A and Phi are determined by the position and velocity of the particle at t = 0. If the starting position is A and t = 0, then phi = 0. Graphically, the equation for SHM is as follows.



In this case, T is the period of motion. Where

$$T = \frac{2\pi}{\omega}$$

The period of motion is defined as the time interval required for the particle to go through a full cycle of its motion. The displacement from equilibrium and the velocity of a particle at time t is the same as the values at t+T.

The inverse of the period of motion is called the frequency (f). Which is the number of full cycles per second.

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

The frequency of SHM is measured in Hertz (Hz).

The period and frequency of SHM of a spring-mass system can be expressed as follows

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Motion equations for SHM:

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = -\omega A sin(\omega t + \phi)$$

$$a = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 A cos(\omega t + \phi)$$

Therefore

$$v_{max} = \omega A$$

$$a_{max} = \omega^2 A$$

Energy in a Simple Harmonic Oscillator

Neglecting friction, the total energy is constant. Meaning that the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 sin^2(\omega t + \phi)$$

And the spring potential energy is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2cos^2(\omega t + \phi)$$

So the total energy would be

$$E = K + U = \frac{1}{2}kA^2$$

In an SHM oscillator system, energy is continuously being transferred between spring potential energy and the kinetic energy of the block.

Energy can be used to find the velocity

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{A^2 - x^2}$$

Phase Angles

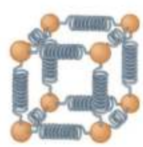
The phase angle phi tells us at what point the cycle the motion was at t = 0.
When t = 0

$$x_0 = Acos(\phi)$$
$$v_0 = -\omega Asin(\phi)$$

The phase angle can be calculated as

$$\phi = cos^{-1}(\frac{x_0}{A})$$

Simple harmonic oscillators are good models for a variety of physical phenomena. For example, If the atoms in a molecule are not too far apart, the forces between them can be modelled a if there were springs between the atoms.



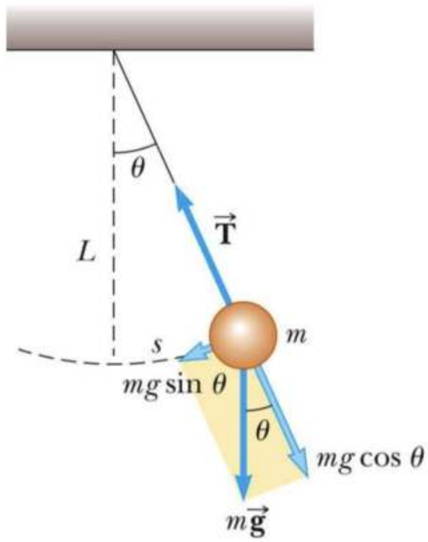
Simple Pendulums

The motion of a simple pendulum is very close to that of a SHM oscillator for angles less than 10 degrees. In the tangential direction, the restoring force is

$$F = -mgsin(\theta)$$

The length L for the pendulum remains constant, and for small values of theta, the displacement s is

$$s = L\theta$$



According to Newton's second law and knowledge of the relationship between tangential and angular acceleration. The restoring force can also be defined as follows

$$F = mL\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

Rearranging the equation

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{L}sin(\theta)$$

Using the small angle approximation

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{L}\theta$$

For a pendulum, the angular frequency is

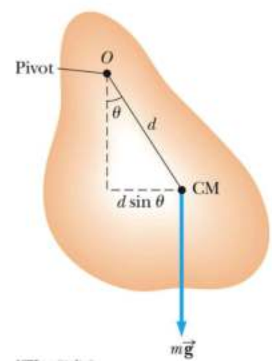
$$\omega = \sqrt{\frac{g}{L}}$$

And the period is

$$T = 2\pi\sqrt{\frac{g}{L}}$$

Physical Pendulums

If an object is hanging about an axis that is not its center of mass, then it cannot be modelled as a particle and a simple pendulum, but a physical pendulum. In this case the gravitational force provides a torque about the axis of rotation O.



The torque can be calculated as follows

$$\vec{\tau} = mgdsin(\theta) = I\alpha = I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

Which means

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{mgdsin(\theta)}{I} = \frac{mgd}{I}\theta$$

We can then deduct

$$\vec{a} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = w^2\theta = -\frac{mgd}{I}\theta$$

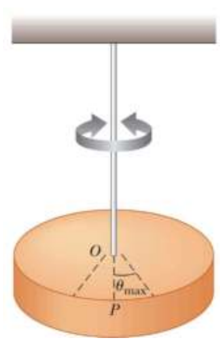
$$\omega = \sqrt{\frac{mgd}{I}} \qquad T = 2\pi\sqrt{\frac{mgd}{I}}$$

Torsional Pendulum

Torsional pendulum is a rigid object suspended from a wire attached to its top at a fixed point. When the object is twisted, the wire would exert a restoring torque that can be defined as follows

$$\vec{\tau} = -\kappa\theta$$

Where Kappa is the torsional constant and theta is the angular displacement.



In this case

$$\vec{\tau} = I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\kappa\theta$$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{-\kappa\theta}{I}$$

This implies

$$\omega = \sqrt{\frac{\kappa}{I}} \quad T = 2\pi\sqrt{\frac{I}{\kappa}}$$

For all pendulums, the SHM equation is

$$\theta(t) = \theta_{max} sin(\omega t + \phi)$$

Waves:

Wave can be defined as a propagation of energy, typically through a medium. There are 2 types of waves.
Mechanical waves are disturbances through some physical medium, such as a string or coil.
Electromagnetic waves do not require a medium to propagate energy, examples of this type of wave are light, radio waves and x-ray.

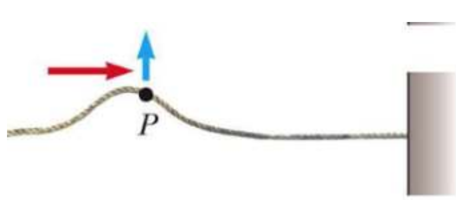
It's important to note that waves do carry mass, but carry energy over a distance.

Requirements for mechanical waves:

- Source of disturbance
- A physical medium
- Elements in the medium can influence each other (Otherwise the energy cannot be propagated)

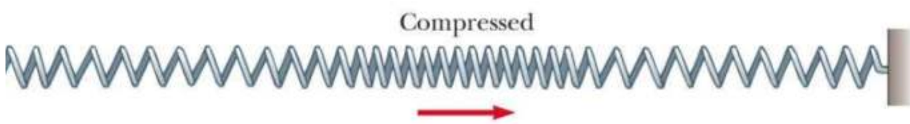
Transverse Waves:

A transverse wave is a wave where the direction motion of an element/particle on the medium moves perpendicular to the direction of propagation. An example of such is waves on a rope.



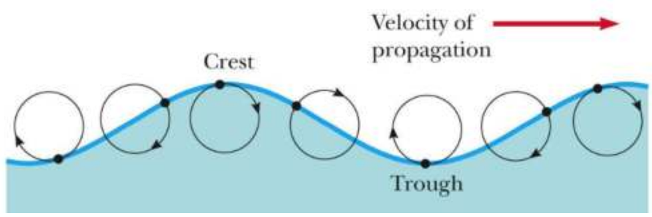
Longitudinal waves:

In contrast to transverse waves, longitudinal waves are present when the movement of elements is parallel to the direction of the propagation of the wave. Note that it does not necessarily have to be in the same direction. An example of longitudinal waves in the wave in a coil.



Complex Waves:

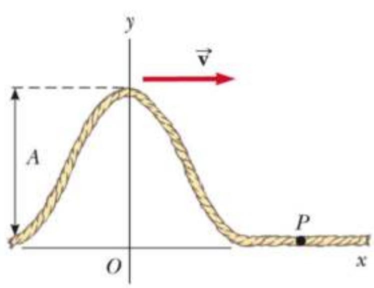
A complex wave is a combination of transverse and longitudinal waves, an example of such are waves in water.



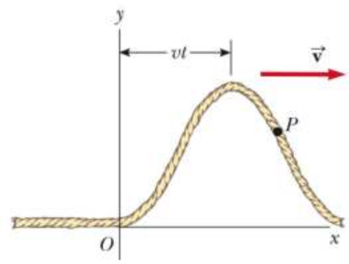
Pulse and the Wave Function:

Pulse travels through a string, and it has definite height and velocity. Its shape does not change much during the propagation.

The pulse at t = 0



The pulse at t

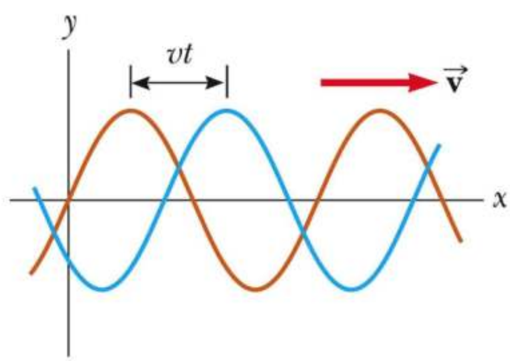


The shape of the pulse at t can be represented by

$$y(x,t) = f(x \pm vt)$$

This function is called the wave function, and it describes the transverse position of an element at point x on the medium at time t. When t is fixed, the wave function is called the wave form.

When the wave is periodic and continuous, it can be represented by a sinusoidal curve.



And the wave function can be represented and follows

$$y(x,t) = Asin(kx - \omega t)$$

The calculation of k and Omega will be shown later.

The Wave Model

The Wave model is a simplification in solving related problems. An ideal wave is assumed, and it has the following characteristics:

- Constant frequency
- Infinite length
- Can be combined with other ideal waves

The lowest point on the wave is called the trough and the highest point the crest.

The wavelength (lambda) is the distance from one trough/crest to the next.
The phase of the wave (kx - wt) changes linearly with time.
Wavelength is not the same as period (T).

Let t = 0, then

$$y(x) = Asin(kx)$$

We know that the displacement y is the same at both ends of a single wavelength, which means

$$x_1 = x + \lambda$$

Thus

$$Asin(kx_1) = Asin[k(x \pm \lambda)] = Asin(kx + k\lambda)$$

Since

$$k\lambda = 2\pi$$

Therefore

$$k = \frac{2\pi}{\lambda}$$

Now, we assume that x = 0. Which means

$$y(t) = Asin(-\omega t) = -Asin(\omega t) = -Asin[\omega(t + T)] = -Asin(\omega t + \omega T)$$

In this case

$$\omega T = 2\pi$$

Therefore

$$\omega = \frac{2\pi}{T}$$

Putting everything together, the wave function can be given by

$$y(x,t) = Asin[2\pi(\frac{x}{\lambda} \pm \frac{t}{T})]$$

It can also be given by

$$y(x,t) = Asin[\frac{2\pi}{\lambda}(x \pm vt)]$$

The speed of traveling waves can be calculated as follows

$$\vec{v} = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} = \lambda f$$

The transverse speed and acceleration of an element at x is

$$v_y = \frac{\mathrm{d}y}{\mathrm{d}t} = -\omega Acos(kx - \omega t)$$

$$a_y = \frac{\mathrm{d}v_y}{\mathrm{d}t} = -\omega^2 Asin(kx - \omega t)$$

The speed of wave propagation is largely affected by the material properties of the medium as well as the tension which the medium is being subjected to.

$$v = \sqrt{\frac{T}{M/L}} = \sqrt{\frac{T}{\mu}}$$

The kinetic energy and potential energy of over 1 wavelength are equal, and can be defined as:

$$K_\lambda = U_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

And the power over 1 wavelength is

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

Interference of Waves:

The superposition of 2 sinusoidal waves can be mathematically represented as follows

$$y'(x,t) = y_1'(x,t) + y_2'(x,t) = Asin(kx - \omega t) + Asin(kx - \omega t + \phi) = [2Acos(\frac{\phi}{2})]sin(kx - \omega t + \frac{\phi}{2})$$

2 sinusoidal waves of the same wavelength and amplitude travelling in opposite directions can result in a special type of wave called a standing wave. Which is represented by

$$y'(x,t) = 2Asin(kx)cos(\omega t)$$

The nodes on a standing wave remain completely stationary, whereas the antinodes experience the highest displacement of 2A, where A is the amplitude of the 2 waves that make up the standing wave. The locations of the nodes and antinode can be found as follows

$$nodes : n\frac{\lambda}{2}$$

$$Antinodes : (n + \frac{1}{2})\frac{\lambda}{2}$$

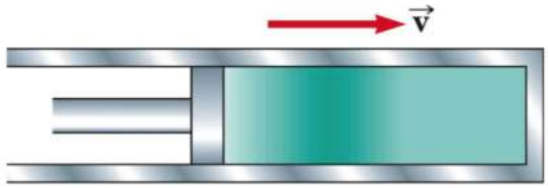
For n = 0, 1, 2, 3, ….

At certain frequencies, standing waves can be created by constructive interference called resonance. The frequency can be defined as follows

$$f = \frac{v}{\lambda} = n\frac{v}{2L}$$

Sound Waves

Sound waves is a type of longitudinal mechanical wave. When a piston moves, it compresses a region of air, and also causes rarefactions to form. This compressed region then travels down the tube not at the speed of piston, but the speed of sound. Sound can travel through any medium, and the speed at which it travels depends on the properties of the medium.



For sound waves travelling through a fluid, the velocity is

$$v = \sqrt{\frac{B}{\rho}}$$

Where B is the bulk modulus of the fluid. For sound waves travelling through a solid medium, the velocity is

$$v = \sqrt{\frac{Y}{\rho}}$$

Where Y is Young's modulus of the solid medium.
The speed of sound also depends on the temperature of the medium.

$$v = 331\sqrt{(1 + \frac{T_c}{273})}$$

Where 331 is the speed of sound at zero degrees Celsius.

Representation of Sound Waves

There are two aspects of sound waves that can be modelled as SH oscillators. First the pressure variation from equilibrium with time t and location x can be defined as

$$\Delta P(x,t) = \Delta P_{max} sin(kx - \omega t)$$

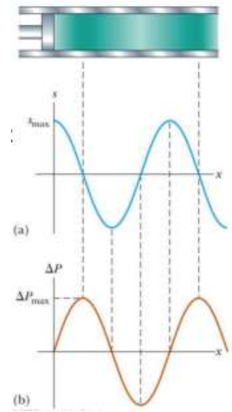
Another aspect is the displacement of the particles of the medium from equilibrium over time. It can be defined as.

$$S(x,t) = S_{max} cos(kx - \omega t)$$

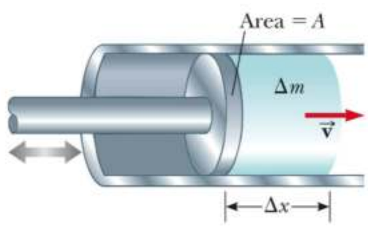
It's worth noting that

$$\Delta P_{max} = \rho v \omega S_{max}$$

As we can see, the sound wave and displacement amplitude is 90 degrees out of phase. Meaning that when the pressure is greatest, the displacement is zero. And when pressure variation is minimum. The displacement is greatest.



Considering a piston creating a sound wave again, the piston transfers energy to the element of air in the tube. And the energy is propagated away in the sound wave. It can be schematically represented as follows.



The kinetic energy over one wavelength is

$$K_\lambda = U_\lambda = \frac{1}{4}\rho A\omega^2 S_{max}^2$$

The power of a sound wave is the amount of energy that goes past a given point during one period of oscillation.

$$P = \frac{1}{2}\rho A v\omega^2 S_{max}^2$$

The intensity of sound in terms of maximum pressure variation is defined as

$$I = \frac{(\Delta P_{max})^2}{2\rho v}$$

The propagation and distribution of sound energy obeys the inverse square law, namely, the intensity of the wave decays as the square of the distance from the energy source. Sound propagates out from the source in a spherical shape, the intensity can be defined as

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

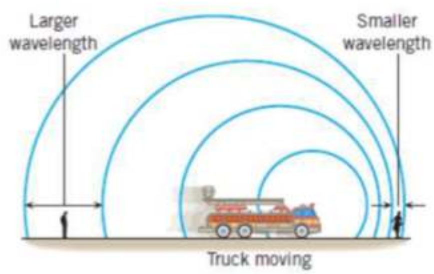
The sound intensity level, measured in DeciBels (dB) is defined as

$$\beta = 10log(\frac{I}{I_0})$$

Where I_0 is called the reference intensity, and I_0 = 1 x 10^-12 W/m^2, otherwise known as the threshold of hearing. The threshold of pain is 1W/m^2, or around 120 dB.

The Doppler Effect

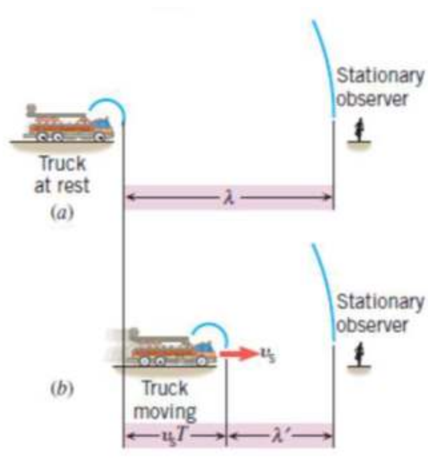
The Doppler effect is the apparent change in frequency of sound waves generated by a source that is not stationary in the reference frame of an observer. Sound waves do not travel faster even when their source is moving. Therefore, the frequency at the front of the moving source appears to be higher than the frequency of sound behind the source. Graphically, it can be represented as follows



The lines that represent the sound waves are called wavefronts.
For sources moving and the observer being stationary, the apparent frequency can be calculated as follows.

$$f_0 = f_s(\frac{1}{1 \pm \frac{v_s}{v}})$$

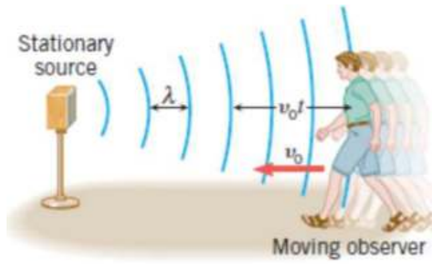
Graphical representation:



For moving observers and the source being stationary, the equation for the apparent frequency is slightly different.

$$f_0 = f_s(1 \pm \frac{v_0}{v})$$

Graphical Representation:

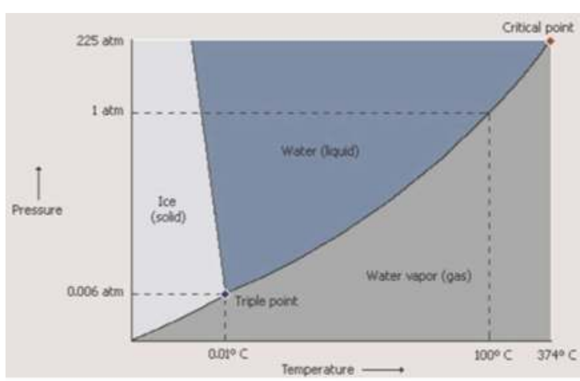


<u>Thermodynamics:</u>

<u>Temperature and Heat:</u>
The technical and formal definition of <u>temperature</u> is that temperature is a property that determines whether or not an object is in <u>thermal equilibrium</u> with another object. If 2 objects are in thermal equilibrium, then their temperature is the same, and no heat energy will be exchanged when they come in <u>thermal contact</u>.

The 0th Law of Thermodynamics:
If object A and B are separately in thermal equilibrium with object C, then A and B are in thermal equilibrium.

Temperature is measured using the Kelvin scale. A standard fixed reference point is chosen, and that point is the <u>triple point of water.</u> Which is where water can exist in all 3 states (vapor, liquid and ice). The temperature of the triple point is 273.16 K, and the pressure is 0.006 atmospheres.



Another method of determining the Kelvin scale is by looking at the pressure of different gases as the temperature decreases. It turns out that all of the pressure of gases converges into a single value at -273.15 degree Celsius. This temperature is otherwise known as <u>absolute zero</u>. Where all molecules cease to have motion. This is the temperature of 0 K.

Thermal Expansion:

As the temperature of an object rises, the average separation of atoms is higher, since there's greater vibrational amplitude. This results in the overall size of the object becoming larger. All the dimensions of an object will increase as it thermally expands.

Linear Expansion:

Suppose an object has an initial length L. Their change in length with respect to change in temperature is as follows.

$$\Delta L = \alpha L_i \Delta T$$
$$L = L_0(1 + \alpha \Delta T)$$

Where alpha is the coefficient of linear expansion.

Volume Expansion:

Linear expansion is only possible for solids, whereas volume expansion; a similar idea, applied to both solids and fluids. We define the change in volume with temperature to be

$$\Delta V = \beta V_0 \Delta T$$
$$V = V_0(1 + \beta \Delta T)$$

Where beta is the coefficient of volume expansion. For an isotropic solid

$$\beta = 3\alpha$$

Heat Capacity and Specific Heat:

Heat capacity is defined as the amount of energy required to raise the temperature of a material by 1 degree celsius. If energy Q is needed to produce Delta T. Then

$$\Delta Q = C\Delta T = C(T_f - t_i)$$
$$C = \frac{\Delta Q}{\Delta T}$$

Here, it's useful to define the term heat. It is the energy that is transferred between a system and its environment due to a temperature gradient.

Heat capacity does not take mass into account, this is where specific heat comes in. If energy Q is transferred to a sample of mass m and causes a change in temperature. Its specific heat is.

$$c = \frac{C}{m} = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$
$$Q = mc(T_f - T_i)$$

In reality, the heat capacity varies with temperature. But these variations are negligible for small values of change in temperature.

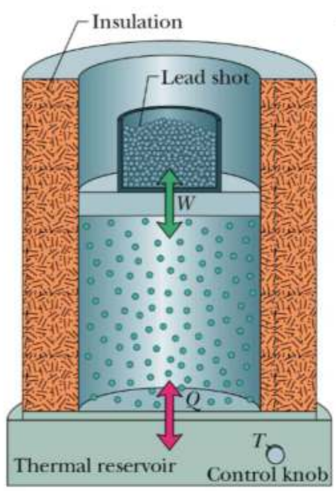
$$Q = \int_{T_i}^{T_f} c dT$$

Heat energy may not necessarily lead to a change in temperature, but may cause a sample to transition from one phase to another. Such as from a solid to liquid or from liquid to a gas. During the phase change, there's no change in the temperature of the sample. The heat energy used solely for the phase change is called the latent heat.
Suppose Q is the energy needed to change the phase of a sample with mass m by temperature Delta T.

$$L \equiv \frac{Q}{m}$$

Where L is the latent heat.

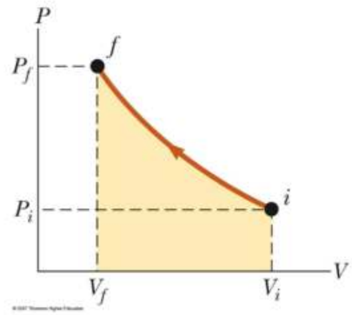
Heat and Work:
A deformable gas is able to do work, and work can also be done on it. This is done by expanding the gas by increasing its temperature. Consider the schematic diagram below.



If a few lead shots are removed from the top, the gas would slowly expand. Because the process is slow. The system remains in thermal equilibrium, and it's said to be quasi-static.
The work done by the gas can be defined as

$$W = -\int_{V_i}^{V_f} p dV$$

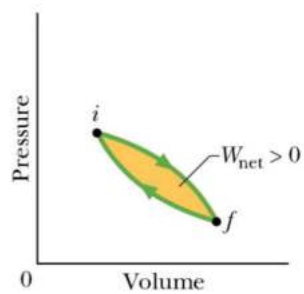
A thermodynamic process is where you start with an initial state with some volume, pressure and temperature, and end up in a final state with a different parameter(s). It can be visualised using the P-V Diagram. As we can also see, the work done by the gas is the area under the curve. And the curve itself is called the path.



Both the work and Heat depends on the nature of the process. However, the quantities Q and W are the same for all processes. It only depends on the initial and final states of the process, but not the process itself. The quantity *Q - W* represents the change of an intrinsic property of a system called its internal energy.

Isolated systems are systems that do not interact with its surroundings, and therefore. $Q = W = 0$, and the internal energy remains constant.

On the other hand, a cyclic system is one that starts and ends in the same state. Its process would not be isolated. And some net work is done by the system. An example of a cyclic system is the internal combustion engines of an automobile.



The First Law of Thermodynamics:

$$\Delta E_{int} = Q - W$$

Where W is the work done by the gas, and Q is the heat added to the system
This is a special case of the conservation of energy that is applied to thermodynamic systems.

The Ideal Gas Law and Boyle's Law:

The interatomic forces within gases are very weak. Also unlike solids and liquids, gases do not have an equilibrium separation for their atoms, therefore there's no standard volume of a gas at some temperature. A gas assumed to be at very low density and also with intermolecular forces being ignored is an ideal gas.

The ideal gas law states that if the volume and temperature of a fixed amount of gas does not change, then its pressure would not change.

$$PV = nRT$$

Boyle's law states that for an ideal gas, its pressure is inversely proportional to the volume.

$$p \propto \frac{1}{V}$$

And hence

$$p_1V_1 = p_2V_2$$

Thermal Processes:

There are 4 main types of thermal processes.

Isochoric Process:

In this type of process, the volume of the gas remains constant. Therefore, the gas did no mechanical work. Using the ideal gas law, the heat in the system is

$$Q = \frac{C_V}{R}V(P_f - P_i)$$
$$Q = m_{cv}\Delta T = C_v n\Delta T$$

Where C_v is the specific heat at a constant volume.

Isobaric Process:

In this type of process, the pressure remains constant. The work done can then be calculated as.

$$W = P\Delta V$$
$$\Delta E_{int} = Q - W = mc_p\Delta T - P\Delta V$$

Adiabatic Process: In this process, there's no heat transfer in or out of the system. Thus

$$\Delta E_{int} = -W$$
$$W = \frac{P_2V_2 - P_1V_1}{1-\gamma}$$

Where gamma is

$$\gamma = \frac{C_p}{C_v}$$

Isothermal Process: In this process, the temperature of the system remains constant. The plot on the PV diagram yields a hyperbolic curve. Any heat energy entering the system must be transferred out to the system via mechanical work. Thus $Q = W$.
The work done in an isothermal process can be calculated as

$$W = nRTln(\frac{V_i}{V_f})$$

Heat Transfer

There are 3 main mechanisms of heat transfer:

- **Conduction** - Transfer of vibrational energy between atoms
- **Convection** - Colder fluid takes away heat.
- **Radiation** - Heat radiated in the form of electromagnetic waves.

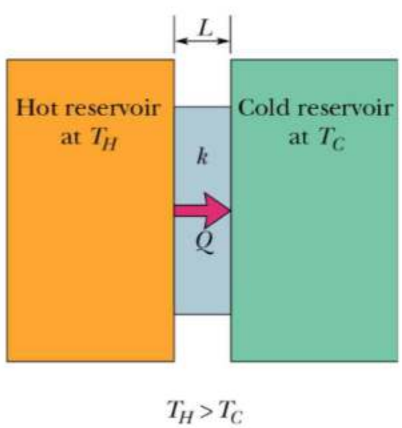
Conduction. The amount of heat Q transferred per unit time can be calculated as

$$P_{conduction} = \frac{Q}{t} = kA\frac{T_H - T_C}{L}$$

Where k is the coefficient of conductivity and A is the area. Another parameter called the R-value, is the thermal resistance to conduction, and it can be calculated by.

$$R = \frac{L}{k}$$

Schematically, heat conduction can be shown below.



Convection: Convection is when the heat reservoir comes in contact with a colder fluid, the fluid gets heated up, expands, and rises. Taking the heat away.

Radiation: In this case, heat is transferred by electromagnetic waves. Radiation requires no medium. The rate at which an object emits energy via radiation is

$$P_{rad} = \sigma \epsilon A T^4$$

Where epsilon is the emissivity.
The rate at which an object absorb thermal radiation from it environment is

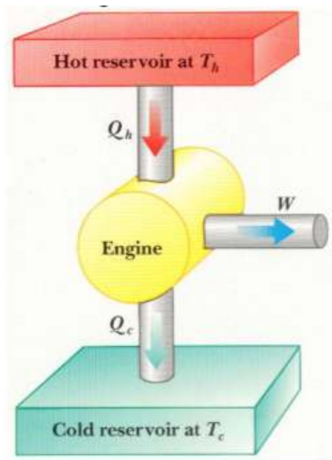
$$P_{absorbed} = \sigma \epsilon A T_{env}^4$$

And the net rate of energy exchange is

$$P_{net} = \sigma \epsilon A (T_{env}^4 - T^4)$$

Heat Engines:

Heat engine is a device that takes energy by heat, and operating in a cyclic process, expels a fraction of that energy by means of work. A schematic can be shown below



In this case Q_h is the energy absorbed by heat from the high temperature energy reservoir, and Q_c is the heat energy expelled to the lower temperature reservoir. The work done can be defined as

$$W = |Q_h| - |Q_c|$$

The thermal efficiency of a heat engine can be calculated as follows

$$e = \frac{W}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$$

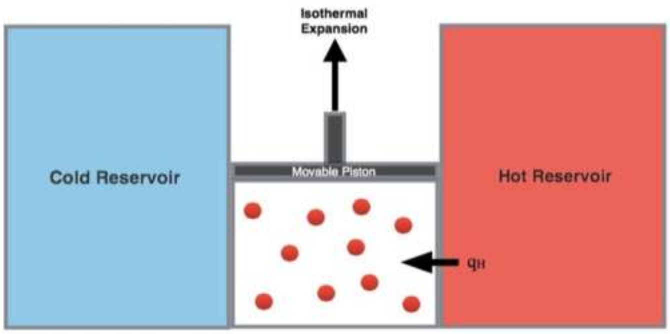
It's impossible for a heat engine to be 100% efficient, some energy from the hot reservoir is bound to be expelled as heat.

The Carnot Engine

The Carnot engine is a theoretical heat engine designed by Sadi Carnot, and it sets an upper bound of the efficiency of heat engines. The carnot engine operates in the carnot cycle, of which each of its processes are reversible and ideal. No engines can operate more efficiently than a Carnot engine, since reversible processes are impossible in reality and the efficiency of real engines are further reduced through friction and energy losses through conduction.

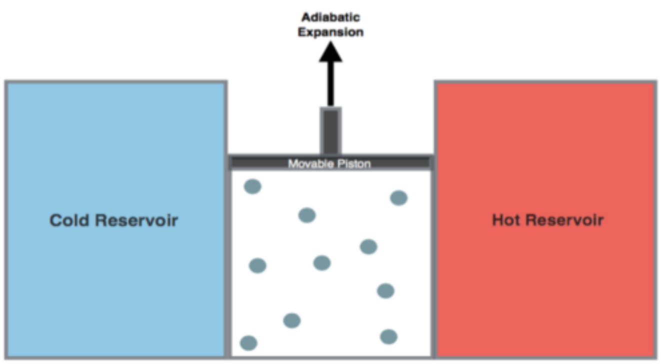
<u>The Carnot Cycle</u>

<u>Stage 1: Isothermal Expansion</u>



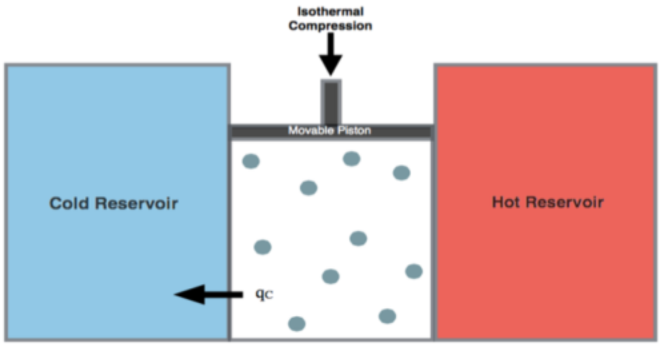
Heat is transferred reversibly from a high temperature reservoir, but the temperature is kept constant by the heat coming in from the hot reservoir. The gas expands, which does work on the piston. The entropy increases by Q_1/T_h.

<u>Stage 2: Adiabatic Expansion</u>



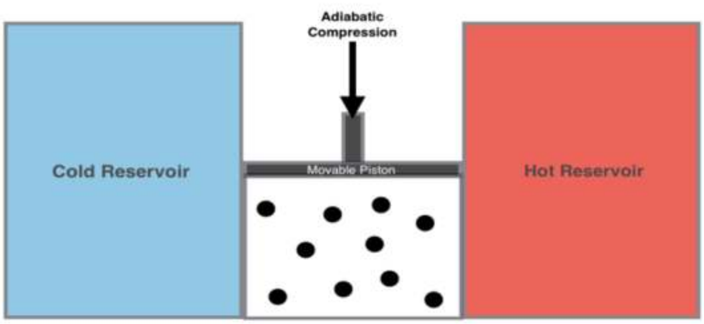
The cold and hot reservoirs are thermally insulated, and thus no heat is gained or lost. The gas continues to expand by reduction of pressure. The entropy remains constant. The expanded gas now has the temperature T_c.

<u>Stage 3: Isothermal Compression</u>



The gas is now in thermal contact with the cold reservoir, and the surroundings now do work on the gas, compressing it. It also causes Q_c amount of thermal energy to leave the gas. The entropy decreases by the same amount as it increases in the first step.

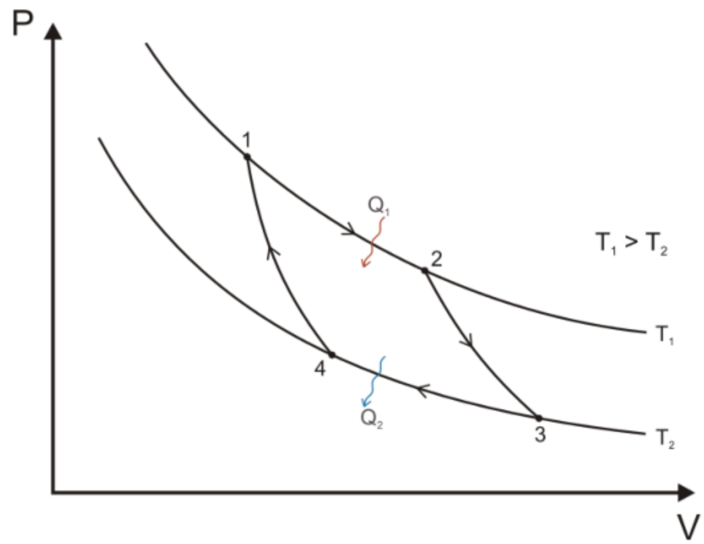
Stage 4: Adiabatic Compression



Once the gas is again insulated from both reservoirs. The surroundings continue to do work on the gas, further compressing it. Causing the temperature to rise back to T_h solely by the work done on the system. But the entropy remains unchanged.

Note: The Carnot engine is assumed to be frictionless.

The P-V diagram of the Carnot cycle is shown below.

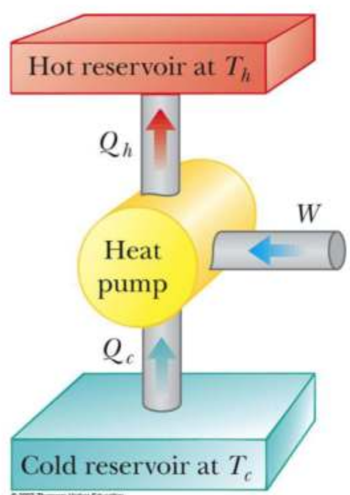


The work done by the engine is the area of the enclosed region. And the net work is Q_h - Q_c. The efficiency of a Carnot engine can be calculated as

$$e = \frac{T_h - T_c}{T_h}$$

<u>Heat Pumps and Refrigerators:</u>

Heat pumps and refrigerators are heat engines in reverse. Where energy is extracted from a cold reservoir and transferred to a hot reservoir. This is not a natural direction of heat transfer, and energy must be put into a device in order to do this.



$$|Q_h| = |Q_c| + W$$

The effectiveness of a heat pump is determined by the <u>coefficient of performance</u> (COP).

$$COP = \frac{|Q_c|}{W} = \frac{|Q_c|}{|Q_H| - |Q_c|}$$

Entropy

The big idea of the second law of thermodynamics is that energy does not transfer spontaneously by heat from a cold reservoir to a hot reservoir. Rather it requires work to be done on it. These processes can only be partly reversed. The direction of these processes are not addressed by the first law of thermodynamics. The second law of thermodynamics states the direction of these processes, and it is set by the change in entropy of the system. When an irreversible process occurs within a closed system, the entropy always increases, all natural processes are irreversible. And here we can conclude the second law of thermodynamics.

The second law of thermodynamics:
The entropy of the universe increases in all real processes.

Entropy itself is not a useful quantity, but the change in entropy is, and it can be calculated as.

$$\Delta S = \int_i^f dS = \int_i^f \frac{dQ}{T} = \frac{\Delta Q}{T}$$

Entropy does not give regard to the intermediate states of change for temperature, pressure and volume of the gas. But only the initial and final states. The entropy change for an irreversible process can be determined by calculating the change in entropy of a reversible process that connects the two states.