A CONCEPTUAL APPROACH TO MATHEMATICS

TEACHERS GUIDE

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<u>LEARNING:</u> is a relatively permanent change in knowledge that occurs as a result of experience.

Learning happens when we can either connect what we has been taught to real life situations (what is already known) or when new concepts are consolidated (compounded over time).

Students learn from their mistakes and thus must be allowed to make mistakes. This therefore means that students must play the central role in their learning. It has been long evident that what we hear we forget, what we see we know and what we use we understand. This means that students who pass their exams are the ones that goes over their work on their own and try to understand it. Yes! there are a few exceptions and some students can recall most of what was taught in class. However, this same student is unable to apply this knowledge unless they can see what is being taught and this only comes through constant self-struggle with concepts and ultimate understanding.

Teaching mathematics for **mastery** is the key to success in the class room. This form of teaching allows the brain to consolidate concepts to memory.

Passive teaching presents procedures in **isolation**. Once a teacher is finish with a topic, the teacher moves on. Teachers normally rely on students to go home and study. However, students in this century do not like to study or do homework. Once a teacher moves on from an isolated topic, the student's brain has been programmed to also move on and forget about what was taught previously in most cases.

It has been evident and you will agree that we can be doing something, following the instructions as told and once we are finish doing it, we will forget about it if it has **no interest** to us. This has been the common scenario in our class rooms. Teachers must understand that to remember something it has to either be interesting to us or our brains have to be commanded to recall the fact, procedure or diagram. It therefore means that in today's mathematics class room, where most topics are not interesting enough, students must be instructed using various techniques to recall.

Our brains recall easily things that are exciting to us such as fights. This is because during these events we are in a heighten state of mental awareness due to increase adrenaline in our bodies which increase oxygen flow and ultimate energy for our brain cells. This event can be forgotten though if this memory is not consolidated. Consolidation of this memory occurs when the person physiologically tell themselves that they want to remember in order to tell others. This briefly places the memory in the long term. Upon telling others about this memory it is compounded and thus remembered for years.

Students remember a song even when they don't want to remember it, because they hear it over and over. Most times the majority of the song is not remembered word for word but fragments of it is. In this cause we can recall but not necessarily understand the song.

This text book presents a completely student centre approach to learning mathematics. Students will be expected to struggle with guidance to learn new concepts. All concepts are inter related. The need for students to recall basic facts, procedure and drawings is addressed. Students are taught to recall through peer to peer explanations and self-instructions to remember by repeating a fact, procedure and drawing a number of times to themselves. Comprehension is taught by developing student's application skills over time through the use of an application guide instrument. Reasoning is taught by developing student's problem solving skills, justification skills, explanation skills and conclusion skills through the use of problem solving, justification, explanation and conclusion instruments to guide student's until they have mastered these skills.

Students must be taught to recall these skills just as they are required to recall facts etc.

STRATEGIES FOR LEARNING ACCORDING TO: https://www.edutopia.org/article/why-students-forget-and-what-you-can-do-about-it

1. **Peer to peer explanation:** When students explain what they've learnt to peers, fading memories are reactivated, strengthened and consolidated.
2. **The spacing effect**: Instead of covering topics and moving on, revisit key ideas throughout the school year. Incorporate a brief review of what was covered several weeks ago.
3. **Frequent practice tests**: Akin to regularly reviewing material, giving frequent practice tests can boost long-term retention and, as a bonus, help protect against stress, which often impairs memory performance.
4. **Interleave concepts**: Instead of grouping similar problems together, mix them up. Solving problems involves identifying the correct strategy to use and then executing the strategy. When similar problems are grouped together, students don't have to think about what strategies to use—they automatically apply the same solution over and over. Interleaving forces students to think on their feet, and encodes learning more deeply.
5. **Combine text with images**: It is easier to remember information that's presented in different ways.

TEACHERS MUST NOTE:

1. Memory needs to sit until it becomes resistant to interferences from competing stimuli. This only happens when we teach for mastery (teach concepts and reasoning with concepts) until students are perfect.
2. If a student cannot verbally repeat a procedure or fact the student **does not** know it.

3. Once teachers move on from non-consolidated concepts, facts or procedures, student's brains also move on (forget it).
4. Once the brain cannot relate to a concept, it will forget it in minutes (this is automatic). Test students by having them explain in their own words after teaching them how to explain in general.
5. If the brain is stuck trying to process a concept, any new concepts or words spoken will not be registered and this student will be blank.
6. Short term memory has a 1 minute spam. If concepts, procedure etc. are not committed to long term memory by then it is lost in most cases.
7. **Since the brain is highly** selective about what it retains for the long term, teachers must do the following or any alternative thereof at the end of each concept in order for students brains to informed that the concept is important:
    a. 1st repetition –Right after learning
    b. 2nd repetition- After 20-30 min
    c. 3rd repetition-After 1 day
    d. 4th repetition-After 2-3 weeks
    e. 5th repetition-After 2-3 months
8. All topics must be taught by inter relating and where possible with familiar applicable situations.
9. Expect your classes to produce good noise.

The teaching methodologies used in this text; cooperative, collaborative and project based learning have all been proven over the years to enhance long term memory.

Too often our teachers are capable of explaining concepts to our students and students seem to understand. However, when asked to explain what they understand, are just incapable of doing so because they don't know what it means to explain and has never been taught to.

Students in lower school are mostly not mature enough to appreciate the rationale behind learning mathematics. It therefore means that students must be taught mathematics by relating it directly to simple relatable items in order to see students learn. In other words most students in non-traditional high schools lack the motivation to learn and only by relating concepts to their current interest will they learn.

Upon reaching grades 10-11. Most students become more mature and therefore are more motivated to learn. At this stage more abstract concepts can be taught.

Teachers should use either deductive or inductive reasoning at the simplest level to facilitate learning:

Deductive reasoning: Give students the rule and have them come up with the solution.

Inductive reasoning: Students create the rule from a few cases.

Relations, Functions, Graphs, Geometry, Trigonometry and Measurements can be taught using a football field and playing or basketball or netball. **This real relatable application** should be used especially for students who lack interest across grades 10-11. These students should be told that they will be taught to use mathematics to better understand the game to enhance chances of winning.

Teachers must always go through the aspect of the book that they will be teaching before class begins.

MATHEMATICS CONCEPTS

The concept of sets is the foundation of mathematics

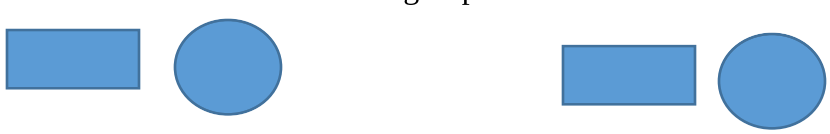
For this text, there will be two kinds of sets; identical sets and non-identical sets. An Identical set will be defined as sets with all its elements being of the same type and a non-identical set will be defined as a set with different type's elements.

Examples of identical sets:

Each set contain one (1) identical element.

Each set contain two (2) identical elements:

Identical sets with of the same group of elements:



Identical sets with different number of elements:

Identical set (s) can contain as many elements as required.

Examples of non-identical sets:

Each set contain one (1) non-identical element

Each set contain two (2) non-identical elements:

Each set is non-identical:

Non-identical sets can contain as many different elements as required.

Note: if the colour and size of the objects in the set(s) are different, then the sets are non-identical.

BASIC ARTITMETIC

ADDITION:

Conceptual Knowledge: Addition asks, how many of the same set of thing(s) or same set of group of thing(s) will you have if you put the individual identical sets or identical groups of each set together. Once these sets are merged, then one larger set is formed of the same identical things or group of things which we can quantify with numbers.

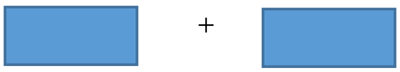
Facts: We cannot conceptually use addition to combine sets that are non-identical to each other. Addition uses the sign (+) which reads ‘PLUS’ or ‘ADD’ or ‘AND’.

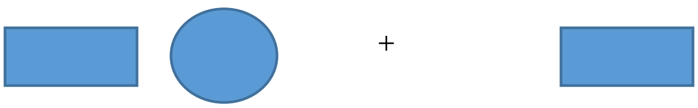
When adding sets with identical elements then each element can be represented by a number for each set. When adding sets of groups, only each group can be represented by a number.

Conceptual Understanding:

This requires you to use the conceptual knowledge and critical thinking to figure out unknowns.

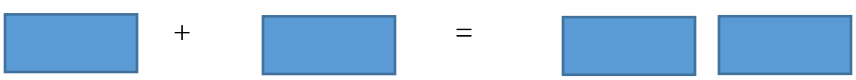
Represent by addition and state the final amount where possible.



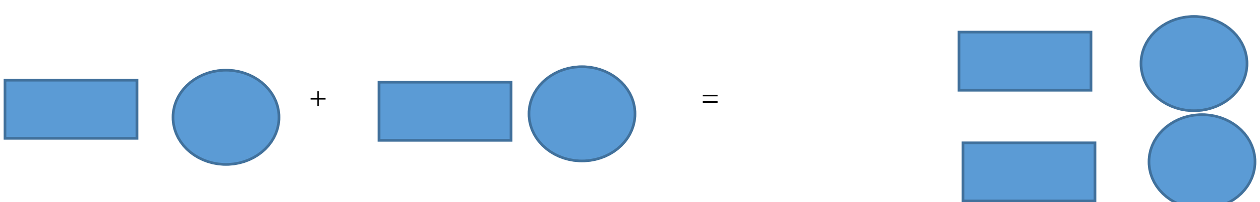


CRITICAL THINKING GUIDE

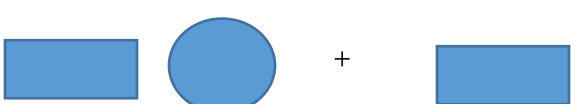
1. Write down what you see
    - What you are trying to find out
    - What you currently know
2. Ask yourself, how does what you are trying to find out compare to what you already know.
3. Ask yourself, how can you fit what you currently know into what you already know about what you are trying to find out



1set of 1 BB 1 set of 1BB same as 1 set of 2 BB or just 2 BB



1 set of 1 group of S 1 set of 1 group of S same as 1 set of 2 groups of S or just 2 S

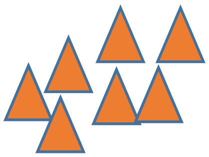
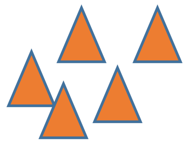


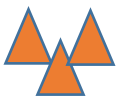
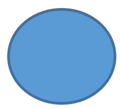
= cannot add because they are non-identical

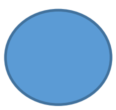
1 set of 1 group of S 1 set of 1 BB

Application:

Using critical thinking and conceptual knowledge determine the following and give a reason for your answer:

 +  =

 +  =

 + 3 =

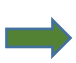
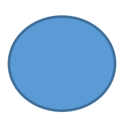
2 + 4 =

CREATIVITY: Create a situation in which addition can be used.

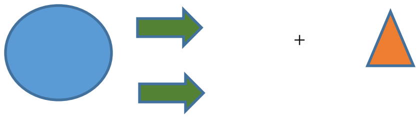
Critical Question #1: If I have multiple items which are different, can't I add them?

Note: You must not confuse union of elements or group of elements with addition. Union means to merge sets of elements to form a larger group of non-identical elements or identical elements.

A number can be used to identify the amount of elements in any type of set. However, once that set is non-identical you can only count the elements from all sets and not add the number of elements of the sets. The result will be a larger numbers with no identity. Again; addition seeks to merge smaller sets of the same elements or same group of elements to form a larger set of those elements or groups.

 +  = we can only count the number of elements to get 2 but not add them to get 1 set of 2.

1 set of 1 A   1 set of 1 C



= we can only count the number of elements to get 4 but not add them to get 1 set of 4.

1 set of 3 1 set of 1 T

4 + 2 = 1 set of 6. These are all numbers so we can add them to create a larger set of numbers.

Critical Question #2. Why my calculator does allow me to use the addition operation to get the answer even when the elements are non-identical?

Note: Your calculator is programed to assume that whatever is being inputted for addition is the same. Since all numbers are considered as identical sets then the calculator will perform the addition operation.

SUBTRACTION

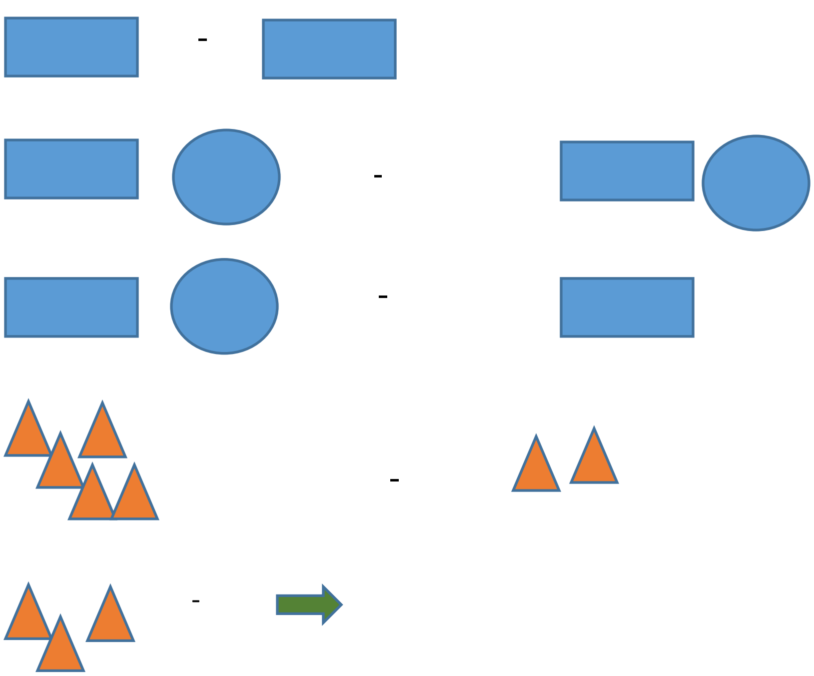
Conceptual knowledge: Subtraction asks, having a set containing the same things or group of things, if you remove an amount of identical set from the set, how many is left.

Facts: Subtraction uses the sign (-) which reads subtract, take away, cancel or remove in most context.

Conceptual Understanding:

This requires you to use conceptual knowledge and critical thinking to figure out unknowns.

Represent by subtraction and state how many is left where possible.



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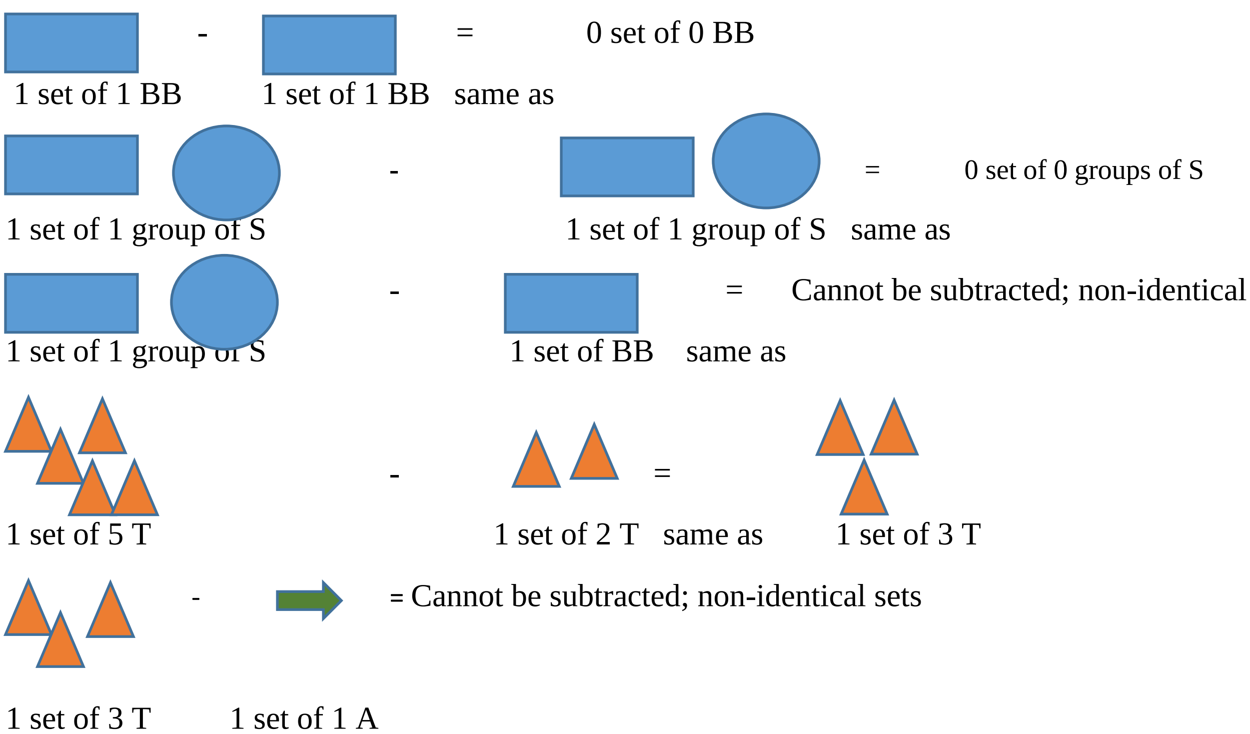


CREATIVITY: Create a situation involving subtraction

Critical Question #1: If I have 5 different objects and I remove one (1) can't I say that one was subtracted?

Note: You must not confuse simply taking away something with subtraction. Subtraction requires that the elements or groups being removed be identical sets. Therefore, you can take way things and not be subtracting them. Having taken away what you want, you can count what is left. Remember, the key to subtraction is that things must be the same.

Critical Question #2. Why my calculator does allow me to use the subtraction operation to get the answer even when the elements are non-identical?

Note: Your calculator is programed to assume that whatever is being inputted for subtraction is the same. Since the objects will be represented by numbers and all numbers are considered as identical sets then the calculator will perform the subtraction operation.

MULTIPLICATION

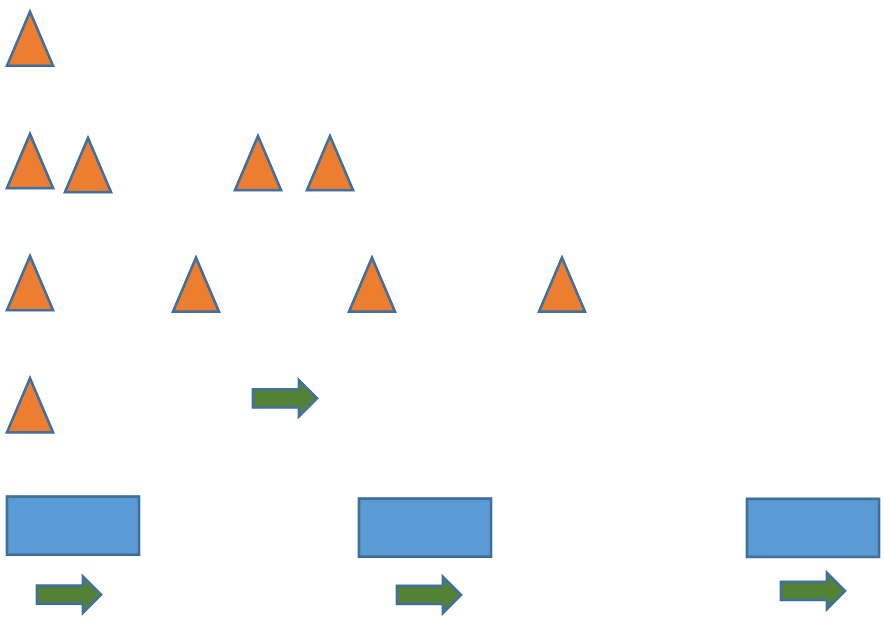
Conceptual Knowledge: Multiplication means “sets of”. It asks, how many sets containing the same amount of elements do we have and how many identical elements or groups will we have. This operation can give rise to both larger and smaller identical sets.

Facts: Multiplication is represented by the sign (x) which reads; multiply, times, of or product. It can be considered repeated addition.

Conceptual Understanding:

This requires you to use conceptual knowledge and critical thinking to figure out unknowns.

Represent by multiplication where possible:



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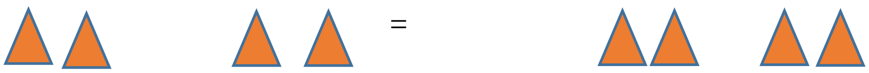
=1 x 1T = 1T

1 set of 1T



=

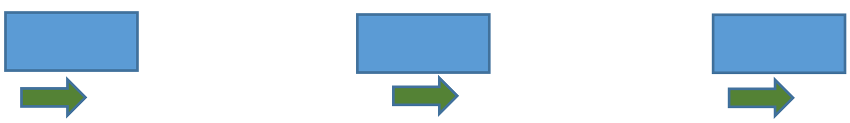
1 set of 2 T   1 set of 2 T   same as   2 set of 2 T or 2 x 2T = 4T

= 4 sets of 1T or 4 x 1T = 4T

1 set of 1 T   1 set of 1 T   1 set of 1 T   1 set of 1 T

= Cannot be multiplied because they are not the same

1 set of 1 T   1 set of 1 A



= 3 set of 1S or 3 x 1S = 3S

1 set of 1 group of S   1 set of 1 group of S   1 set of 1 group of S

CREATIVITY: Create a real life situation to express multiplication.

Critical Question #1: If I have 10 different elements, can't I multiply them?

Note: You can only use multiplication to express identical sets with the same amount of elements. Where the sets are non-identical, you can only count the amount of elements and express it as a number without any identity.

DIVISION

Conceptual Knowledge: Division asks, how many identical sets containing the amount in the divisor is in the dividend for numbers and/or what can be multiplied by the divisor to get the dividend.

Facts: Division is represented by the sign (÷ or /) and reads divide, over or upon. Division can give rise to bigger or smaller amounts.

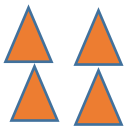
The divisor is the element to the right of the division sign and the dividend is to the left

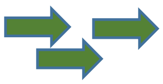
Conceptual Understanding:

This requires you to use conceptual knowledge and critical thinking to figure out unknowns.

Divide the following and find the answer where possible.

 ÷ 

 ÷ 

 ÷ 

6 ÷ 2

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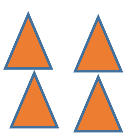
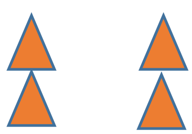
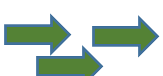
 ÷  = 

1 set of 1T 1 set of 1 T 1 set and we can multiply 1 by the divisor to get dividend is

 ÷  = 

1 set of 4 T 1 set of 2T 2 sets and the number is 2

 ÷  = cannot be divided; non-identical sets

1 set of 3 1 set of 1

6 ÷ 2 = 3 we can multiply 2 the divisor by 3 to get 6

CREATIVITY: Create a real life situation to express division.

Critical Question #1: When can I not use division?

Note: Division cannot be used when the divisor is a non-identical set to the dividend. If you're using numbers, then both the dividend and the divisor must be numbers. If you're using objects or symbols, then the same applies.

INDICES AND EXPONENTS BASICS:

Conceptual Knowledge: Every number or object as you have seen can be expressed as 1 set of that number or object for example 1 is the same as 1 x 1 or 3 can be expressed as 1 X 3 which reads one set of 3. If we want to say "multiply a number by 1 a certain number of times," we can express it mathematically using indices.

Facts: Indices is repeated multiplication. The number we want to multiply be called the base number and the amount of time we want to multiply it by one is the exponent which is represented with a superscript number. For example 5^3 = 1x 5x 5 x 5; this reads multiply five by one and itself three times. The exponent can be

Rational: This is just another way of expressing a mathematical statement it saves time and space.

BASE 10

In mathematics in the Caribbean, Americas and other parts of the world, the base 10 system of numbers is what is used to represent currency and other day to day number representation. We will define face value as the actual numeral and number.

In the base 10 system which gave rise to our place value system, the number 10 is the number base. There are two types of numbers in our place value system; integers and decimals.

Integers are those negative and positive numbers including zero.

Conceptual Knowledge: To derive our place value system, we begin with 10^0 which is 1. This represents the first and smallest whole number place in our place value system; the ONES place. We then have 10^1 which is 10. This represent our second place ranging from smallest to largest which is 10 times the value of the ones place; the Tens Place. We then have 10^2 which is 100. This represent our third place which is 10 times the value of the 10s place; the Hundred Place. We then have 10^3 which is 1000. This represents our fourth place which is 10 times the value of the 100s place. The place value system continues infinitely.

Facts: In the base 10 system, there are ten numbers; 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each place can have maximum face value of up to 9.

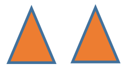
Conceptual Understanding:

THOUSANDS (1X10X10X10)	HUNDREDS (1X10X10)	TENS (1X10)	ONES (1)

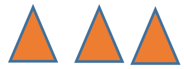
Represent the following number of objects in the place value system.

This requires you to use conceptual knowledge and critical thinking to figure out unknowns.

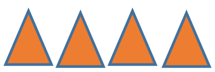
1 set of 1



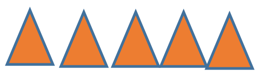
1 set of 2



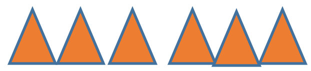
1 set of 3



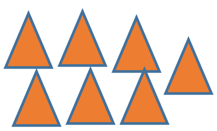
1 set of



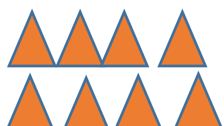
1 set of 5



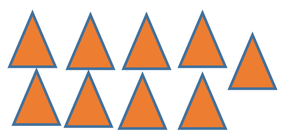
1 set of 6

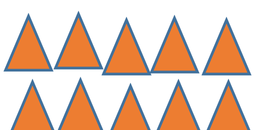


1 set of 7

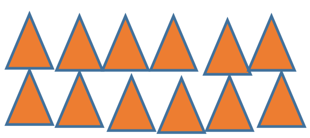


1 set of 8





1 set of 10



1 set of 12

CRITICAL THINKING GUIDE

1. Write down what you see
    - What you are trying to find out
    - What you currently know
2. Ask yourself, how does what you are trying to find out compare to what you already know.

3. Ask yourself, how can you fit what you currently know into what you already know about what you are trying to find out

Critical Question#1: If I have 37 sets of objects do I represent this as 37 ones?

Note: The base 10 system of numbers only allow ten numbers in each place 0-9. Once you have more than nine (9) ones, it must be regrouped and represented as sets of tens. In this case you would have 3 sets of 10s and 7 sets of 1.

Critical Question #2: What about if I have 101 sets of objects, do I represent this as 101 ones?

Note: As stated before, the base 10 system only allow ten numbers in each place 0-9. The objects have to be regrouped in different sets to suit the place value system of numbers. In this case we would have 10 sets of 10 which would give 1 hundred and 1 ones.

NAMING NUMBERS

Conceptual Knowledge: Numbers are named based on the amount of set in each place value.

Illustration #1: 25.. This number has two sets of tens and 5 sets of 1s. This number conceptually is called Two Tens and five ones. Because two tens is also called twenty and five ones is also called five. This number can be called Twenty Five or Twenty and Five.

Illustration #2: 115.. This number has one set of hundred, one set of ten and five sets of ones. This number conceptually is called One Hundred, One Ten and Five Ones.

Because one ten is also and five ones is also called fifteen. This number can be called One Hundred and Fifteen.

Remember Always use your critical thinking guide to help you get to the answer.

BASIC OPERATIONS PROCEDURE ON NUMBERS

ADDITION:

Memory: We can only add the same things or group of things.

Procedural Concept: If we have one set of one and one set of ten (1 & 10) and are asked to add them, we would have to; add the numbers according to the sameness of their place value and set identity.

10 + 1 = 11 in this exercise, 1 set of 10 has 0 ones and 1, 10. 1 set of 1 is just 1 ones. Following procedural concept, we can add zero (0) ones and one (1) ones. This can then be added with 1 set of 10 to give eleven (11).

$$\begin{array}{r} 10 \\ +\ 1 \\ \hline 11 \end{array}$$

Conceptual Understanding indicates that we can add these two numbers because they are both numbers and the addition of the two identical sets will give rise to a larger set of numbers.

Illustration # 2. Twenty five (25) plus one hundred (100). We must always use critical thinking to help us figure out answers.

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Thinking Process:

I am trying to find out how many objects will I have in a larger set if I add two sets.

I know that each set is identical; they are numbers. One set has two sets of tens and 5 sets of ones. The other set has 1 set of hundred.

This compares to the addition of identical sets which requires the addition of things that are the same, like the place value.

To answer this question, what I will do is add the amount of each identical place value for the two sets first and then create a larger set of the two numbers and regroup where necessary.

$$\begin{array}{r} 100 \\ \underline{+25} \\ 125 \end{array}$$

This number is called one hundred and twenty five.

Reasoning/Analysis: 5 sets of ones added to 0 sets of ones produces only 5 sets of 1s. 2 sets of 10s added to 0 sets of 10s produces 2 sets of 10s. 1 set of 100s is added to 0 sets of 100s this give rise to 1 set of 100. The addition therefore produces 125.

Illustration #3: Sixty eight (68) plus fifty six (56). We will use the thought process of critical thinking to figure this out.

Thinking Process:

I am trying to find out how many objects will I have in a larger set if the two sets are added.

I know that each set is identical; they are both numbers. One set has six sets of ten and eight sets of ones. The other set has five sets of ten and six sets of ones.

This compares to the addition of identical sets. This requires the addition of things which are the same, like place vale and obeying the base 10 number rule.

To add these what I would do is to add the same face values of the place values from the different sets first and where necessary regroup sets.

$$\begin{array}{r} 68 \\ \underline{+56} \\ 124 \end{array}$$

Reasoning/Analysis: six sets of ones added to eight sets of ones produce 14 ones. According to the base 10 system 14 ones cannot be placed in the ones column and therefore must be regrouped to produce 1 set of 10 and 4 ones. The one set of 10 now be represented in the 10s column which when added to 6 sets of 10 and 5 sets of 10s, produce 12 sets of 10s. Obeying the base 10 system rule, the 12 sets of tens will be regrouped to produce 1 set of 100s and two sets of 10. This gave rise to the answer 124.

SUBTRACTION

Memory: Subtraction requires the sets to be identical and is just the removal of a set of numbers or objects from an identical set.

Procedural Concept: Like addition you can only subtract from identical sets. Following the sameness concept, subtraction will be done from the identical place values. Where necessary, sets can be regrouped.

Illustration #1: 55 subtract 11. We will use the thought process of critical thinking to figure this out.

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Thinking Process:

We are trying to find out how many of the identical sets will we have left after some is removed.

We currently know that we have 5 sets of 10s and 5 sets of ones and we want to remove 1 set of 10s and 1 set of 1s.

This compares to the removal of identical sets, where we can subtract identical sets of place value amounts.

To do this, we will subtract identical sets place value amounts and regroup the sets where necessary to find out what is left.

$$\begin{array}{r} 55 \\ \underline{-11} \\ 44 \end{array}$$

Reasoning/Analysis: 5 sets of 1s subtract 1 set of 1s produce 4 sets of 1s. 5 sets of 10s subtract 1 set of 10s produces 4 sets of 10s. This leaves 44.

Illustration #2: 31 subtract 16.

Thinking Process:

We are trying to find out how many of the identical sets will we have left after some is removed.

We currently know that we have 3 sets of 10s and 1 sets of ones and we want to remove 1 set of 10s and 6 set of 1s.

This compares to the removal of identical sets, where we can subtract identical sets of place value amounts.

To do this, we will subtract identical sets place value amounts and regroup the sets where necessary to find out what is left.

$$\begin{array}{r} 31 \\ \underline{-16} \\ 15 \end{array}$$

Reasoning/Analysis: We are trying to subtract 6 sets of 1s from 1 set of 1s. This is possible but at this level, we will assume that we can't. We will therefore have to regroup the 3 sets of 10s and make it 2 sets of 10s and 10 sets of 1s. This will allow for a total of 11 sets of 1s and we can now subtract 6 sets of 1s to get 5. We can now subtract the 1 set of 10s from the remainder 2 sets of 10s to get 1 set of 10s. This leaves a total of 15 after the subtraction.

MULTIPLICATION

Memory: Multiplication means sets of. We can only multiply things that are the same and we can also only express the addition of identical sets with same composition as multiplication.

Procedural Concept: Following the concept of identical sets and the sameness concept, we can find sets of a particular place value and where necessary regroup sets to be in-line with the base 10 system.

Illustration #1: 2 x 10. We will apply critical thinking to determine this answer.

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Thinking Process:

We are trying to find out how many of the identical elements are in 2 sets of 10.

We currently know that we have 1 sets of 10s and 0 sets of ones and we want to find out how many elements are in 2 sets of 1, 10s plus 2 sets of 0, 1s.

This compares to the addition of identical sets with exactly the same composition, where we can multiply sets of place value amounts.

To do this, we will find out how many individual elements are in 2 sets of 1, 10s and add this to the amount of elements in 2 sets of 0 ones. We will regroup where necessary

$$\begin{array}{r} 10 \\ \underline{\text{X } 2} \\ 20 \end{array}$$

Reasoning/Analysis: 2 sets of 0, 1s have 0, 1s in total. 2 sets of 1 set of 10s produce 2, 10s. This gave the answer of 20.

Illustration #2: 4 x 23.

Thinking Process:

We are trying to find out how many of the identical elements are in 4 sets of 23.

We currently know that we have 2 sets of 10s and 3 sets of 1s and we want to know how many elements are in 4 sets of 2, 10s added to 4 sets of 3, 1s.

This compares to the addition of identical sets with exactly the same composition, where we can multiply sets of place value amounts.

To do this, we will find out how many individual elements are in 4 sets of 2, 10s and add this to the amount of elements in 4 sets of 3 ones. We will regroup where necessary

$$\begin{array}{r} 23 \\ \underline{X\ 4} \\ 92 \end{array}$$

Reasoning/Analysis: 4 sets of 3, 1s produces 12 sets of 1s. Obeying the base 10 rule, we must regroup the 12 sets of ones to get 1 set of 10s and 2 sets of 1s. We will then find out how many elements are in 4 sets of 2 sets of 10s. This produces 8 sets of 10s. When we add this to the additional 1 set of ten obtained by regrouping, we now have 9 sets of 10s. The multiplication will then produce 92 elements.

Illustration #3: 25 x 31.

Thinking Process:

We are trying to find out how many of the identical elements are in 25 sets of 31.

We currently know that we have 3 sets of 10s and 1 sets of 1s and we want to know how many elements are in 20 sets of 3, 10s added to 20 sets of 1, 1s added to 5 sets of 3, 10s and 5 sets of 1, 1s.

This compares to the addition of identical sets with exactly the same composition, where we can multiply sets of place value amounts.

To do this, we will find out how many individual elements are in 25 sets of 3, 10s and add this to the amount of elements in 25 sets of 1 ones. We will regroup where necessary

$$\begin{array}{r} 31 \\ \underline{X25} \\ 775 \end{array}$$

Reasoning/Analysis: We can begin to work this problem from any direction. At the end we will just be adding the number of elements. We can start by finding 5 sets of 1, 1s which produces 5 elements. We can then find 5 sets of 3, 10s which produces 150(1 set of 100s, 5 sets of 10s and 0 sets of 1s). This produces 155 elements in total.

We will then find 20 sets 1, 1s which produces 20 elements and add that to 20 sets of 3, 10s; which produces 600 elements. When we add the two identical sets we get 620 elements.

Combining the two parts of the operation we get total identical sets of elements are 775.

Critical Question #1: What happens when a set of 10 is multiplied by a set of 1s or a set of 100s multiplied by a set of 10?

Note: Once a set of 10 is multiplied by a set of 1s, it will mostly produce larger sets of 10s. Except when 10, 1s is multiplied by a set of 10s which will produce 100s.

When a set of 100s is multiplied by a set of 10s. It will produce sets larger sets of 100s with the exception of when 100 is multiplied by 10 sets of 10s. Which produces 1000s.

Critical Question #2: What is the difference between addition and multiplication?

Note: You would recall that in addition, the identical sets don't have the same composition/number of elements. In multiplication, the identical sets have to have the same number of elements/composition to be expressed as multiplication.

DIVISION

Memory: Division require that the sets are identical. Division asks the question; how many of the divisor is in the dividend and what can be multiplied by the divisor to get the dividend.

Procedural Concept: In division we want to know how many identical sets containing the same amount in the divisor is in the dividend. This is done for each place value for numbers.

MEASUREMENT

Conceptual Knowledge: When asked to measure something, you are being asked; how many sets of a predetermined object can fit along the stated space.

Facts: Measurements are mostly recorded in millimetres, centimetres, inches, meters, kilometres and miles.