Exercise 4.1

Q.1 Identify whether the following algebraic expressions are polynomials (Yes or No).

(i) $3x^2+\frac{1}{x}-5$

No (Because of $\frac{1}{x}$) Ans.

(ii) $3x^3-4x^2-x\sqrt{x}+3$

No (Because $\sqrt{x}$ or $(x)^{\frac{1}{2}}$) Ans.

(iii) $x^2-3x+\sqrt{2}$

Yes (Because no variable has power in fraction). **Ans**

(iv) $\frac{3x}{2x-1}+8$

No (Because of $\frac{1}{2x-1}$) **Ans**

Q.2 State whether each of the following expressions is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$

Irrational **Ans**

(ii) $\frac{x^3-2x^3+\sqrt{3}}{2+3x-x^2}$

Rational **Ans**

(iii) $\frac{x^2+6x+9}{x^2-9}$

Rational **Ans**

(iv) $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$

Irrational **Ans**

Q.3 Reduce the following expression to the lowest form.

(i) $\frac{120x^2y^3z^5}{30x^3yz^2}$

Solution: $\frac{^4\cancel{120}x^2y^3z^5}{\cancel{30}x^3yz^2}$

$=\frac{120x^2y^3z^5}{30x^3yz^2}$

$=4x^{2-3}y^{3-1}z^{5-2}$

$=4x^{-1}y^2z^3$

$=\frac{4y^2z^3}{x}$ **Ans**

(ii) $\frac{8a(x+1)}{2(x^2-1)}$

Solution: $\frac{8a(x+1)}{2(x^2-1)}$

$=\frac{^4\cancel{8}a(x+1)}{\cancel{2}(x^2-1)}$

$=\frac{4a\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$

$=\frac{4a}{x-1}$ **Ans**

(iii) $\frac{(x+y)^2-4xy}{(x-y)^2}$

Solution: $\frac{(x+y)^2-4xy}{(x-y)^2}$

$\therefore (x+y)^2=x^2+y^2+2xy$

$\therefore (x-y)^2=x^2+y^2-2xy$

$=\frac{x^2+y^2+2xy-4xy}{x^2+y^2-2xy}$

$=\frac{x^2+y^2-2xy}{x^2+y^2-2xy}$

$$= \frac{\cancel{(x-y)^2}}{\cancel{(x-y)^2}}$$

$$= 1 \textbf{ Ans}$$

(iv) $\dfrac{\left(x^3-y^3\right)\left(x^2-2xy+y^2\right)}{(x-y)\left(x^2+xy+y^2\right)}$

Solution: $\dfrac{\left(x^3-y^3\right)\left(x^2-2xy+y^2\right)}{(x-y)\left(x^2+xy+y^2\right)}$

$$\left(a^3+b^3\right) = (a-b)\left(a^2+ab+b^2\right)$$

$$= \frac{\cancel{\left(x^3-y^3\right)}\left(x^2-2xy+y^2\right)}{\cancel{\left(x^3-y^3\right)}}$$

$$= x^2-2xy+y^2$$

$$\therefore (x-y)^2 = x^2-2xy+y^2$$

$$= (x-y)^2 \textbf{ Ans}$$

(v) $\dfrac{(x+2)\left(x^2-1\right)}{(x+1)\left(x^2-4\right)}$

Solution: $\dfrac{(x+2)\left(x^2-1\right)}{(x+1)\left(x^2-4\right)}$

$$= \frac{(x+2)\left[(x)^2-(1)^2\right]}{(x+1)\left[(x)^2-(2)^2\right]}$$

$$= \frac{\cancel{(x+2)}\,(x-1)\,\cancel{(x+1)}}{\cancel{(x+1)}\,(x-2)\,\cancel{(x+2)}}$$

$$= \frac{(x-1)}{(x-2)} \textbf{ Ans}$$

(vi) $\dfrac{x^2-4x+4}{2x^2-8}$

Solution: $\dfrac{x^2-4x+4}{2x^2-8}$

$$\therefore (a-b)^2 = a^2-2ab+b^2$$

$$\therefore a^2-b^2 = (a+b)(a-b)$$

$$= \frac{(x)^2-2(x)(2)+(2)^2}{2\left(x^2-4\right)}$$

$$= \frac{(x-2)^2}{2\left[(x)^2-(2)^2\right]}$$

$$= \frac{(x-2)^2}{2(x+2)(x-2)}$$

$$= \frac{(x-2)\,\cancel{(x-2)}}{2(x+2)\,\cancel{(x-2)}}$$

$$= \frac{x-2}{2(x+2)} \textbf{ Ans}$$

(vii) $\dfrac{64x^5-64x}{\left(8x^2+8\right)(2x+2)}$

Solution: $\dfrac{64x^5-64x}{\left(8x^2+8\right)(2x+2)}$

$$= \frac{64x\left(x^4-1\right)}{8\left(x^2+1\right).2(x+1)}$$

$$= \frac{64\left[\left(x^2\right)^2-(1)^2\right]}{16\left(x^2+1\right)(x+1)}$$

$$= \frac{^4\cancel{64}\left(x^2-1\right)\cancel{\left(x^2+1\right)}}{\cancel{16}\cancel{\left(x^2+1\right)}(x+1)}$$

$$= \frac{4x(x-1)\,\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$= 4x(x-1) \textbf{ Ans}$$

(viii) $\dfrac{9x^2-\left(x^2-4\right)^2}{4+3x-x^2}$

Solution: $\dfrac{9x^2-\left(x^2-4\right)^2}{4+3x-x^2}$

$$= \frac{(3x)^2 - (x^2-4)^2}{4+3x-x^2}$$

$$= \frac{(3x+x^2-4)(3x-x^2+4)}{4+3x-x^2}$$

$$= \frac{(x^2+3x-4)\cancel{(-x^2+3x+4)}}{\cancel{(-x^2+3x+4)}}$$

$$= x^2+3x-4 \textbf{ Ans}$$

Q.4 Evaluate

(a) $\frac{x^3y-2z}{xz}$ **for**

(i) $x=3, y=-1, z=-2$

(ii) $x=-1, y=-9, z=4$

Solution for 1st part

When $x=3,\ y=-1,\ z=-2$

$$\frac{x^3y-2z}{xz} =$$

$$= \frac{(3)^3(-1)-2(-2)}{(3)(-2)}$$

$$= \frac{27(-1)+4}{-6}$$

$$= \frac{-27+4}{-6}$$

$$= \frac{-23}{-6}$$

$$= \frac{23}{6} \textbf{ Ans}$$

Solution for 2nd Part.

When $x=-1,\ y=-9,\ z=4$

$$\frac{x^3y-2z}{xz} =$$

$$= \frac{(-1)^3(-9)-2(4)}{(-1)(4)}$$

$$= \frac{-1(-9)-8}{-4}$$

$$= \frac{9-8}{-4}$$

$$= \frac{1}{-4}$$

$$= -\frac{1}{4} \textbf{ Ans}$$

(b) $\frac{x^2y^2-5z^4}{xyz}$ for $x=4, y=-2$ and $z=-1$

Solution: $\frac{x^2y^3-5z^4}{xyz}$

$$= \frac{(4)^2(-2)^3-5(-1)^4}{(4)(-2)(-1)}$$

$$= \frac{16(-8)-5(1)}{8}$$

$$= \frac{16(-8)-5(1)}{8}$$

$$= \frac{-128-5}{8}$$

$$= -\frac{133}{8}$$

$$= -16\frac{5}{8} \textbf{ Ans}$$

Q.5 Perform the indicated operation and simplify.

(i) $\frac{15}{2x-3y} - \frac{4}{3y-2x}$

Solution: $\frac{15}{2x-3y} - \frac{4}{3y-2x}$

$$= \frac{15}{2x-3y} - \frac{4}{-2x+3y}$$

$$= \frac{15}{2x-3y} - \frac{4}{-(2x-3y)}$$

$$= \frac{15}{2x-3y} + \frac{4}{2x-3y}$$

$$= \frac{19}{2x-3y} \textbf{ Ans}$$

(ii) $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$

Solution: $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$

$$=\frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$$

$$=\frac{(1)^2+(2x)^2+2(2x)(1)-\left[(1)^2+(2x)^2-2(2x)(1)\right]}{(1)^2-(2x)^2}$$

$$=\frac{1+4x^2+4x-\left[1+4x^2-4x\right]}{1-4x^2}$$

$$=\frac{1+4x^2+4x-1-4x^2+4x}{1-4x^2}$$

$$=\frac{4x+4x}{1-4x^2}$$

$$=\frac{8x}{1-4x^2}$$ **Ans**

(iii) $\dfrac{x^2-25}{x^2-36}-\dfrac{x+5}{x+6}$

Solution: $\dfrac{x^2-25}{x^2-36}-\dfrac{x+5}{x+6}$

$$=\frac{(x)^2-(5)^2}{(x)^2-(6)^2}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)}{(x+6)(x-6)}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)-(x-6)(x+5)}{(x+6)(x-6)}$$

$$=\frac{(x+5)\left[(x-5)-(x-6)\right]}{x^2-6^2}$$

$$=\frac{(x+5)(x-5-x+6)}{x^2-36}$$

$$=\frac{(x+5)(1)}{x^2-36}$$

$$=\frac{x+5}{x^2-36}$$ **Ans**

(iv) $\dfrac{x}{x-y}-\dfrac{y}{x+y}-\dfrac{2xy}{x^2-y^2}$

Solution: $\dfrac{x}{x-y}-\dfrac{y}{x+y}-\dfrac{2xy}{x^2-y^2}$

$$=\frac{x(x+y)-y(x-y)}{(x-y)(x+y)}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+\cancel{xy}-\cancel{xy}+y^2}{(x)^2-(y)^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2}{x^2-y^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2-2xy}{x^2-y^2}$$

$$=\frac{(x-y)^2}{x^2-y^2}$$

$$=\frac{(x-y)\cancel{(x-y)}}{(x+y)\cancel{(x-y)}}$$

$$=\frac{x-y}{x+y}$$ **Ans**

(v) $\dfrac{x-2}{x^2+6x+9}-\dfrac{x+2}{2x^2-18}$

Solution: $\dfrac{x-2}{x^2+6x+9}-\dfrac{x+2}{2x^2-18}$

$$=\frac{x-2}{(x)^2+2(3)(x)+3^2}-\frac{x+2}{2(x^2-9)}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2\left[(x)^2-(3)^2\right]}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2(x-3)(x+3)}$$

$$=\frac{x-2}{(x+3)(x+3)}-\frac{x+2}{2(x+3)(x-3)}$$

$$=\frac{2(x-3)(x-2)-(x+3)(x+2)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-2x-3x+6)-(x^2+2x+3x+6)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-5x+6)-(x^2+5x+6)}{2(x+3)(x+3)(x-3)}$$

$$= \frac{2x^2 - 10x + 12 - x^2 - 5x - 6}{2(x+3)^2(x-3)}$$

$$= \frac{x^2 - 15x + 6}{2(x+3)^2(x-3)} \textbf{ Ans}$$

(vi) $\dfrac{1}{x-1} - \dfrac{1}{x+1} - \dfrac{2}{x^2+1} - \dfrac{4}{x^4-1}$

Solution: $\dfrac{1}{x-1} - \dfrac{1}{x+1} - \dfrac{2}{x^2+1} - \dfrac{4}{x^4-1}$

$$= \frac{(x+1)-(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{x}+1-\cancel{x}+1}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{2x^2}+2-\cancel{2x^2}+2}{(x^2)^2-(1)^2} - \frac{4}{x^4-1}$$

$$= \frac{4}{x^4-1} - \frac{4}{x^4-1}$$

$$= \frac{4-4}{x^4-1}$$

$$= \frac{0}{x^4-1}$$

$= 0$ **Ans**

Q.6 Perform the indicated operation and simplify.

(i) $(x^2-49).\dfrac{5x+2}{x+7}$

Solution: $(x^2-49).\dfrac{5x+2}{x+7}$

$$= \left[(x)^2-(7)^2\right].\frac{5x+2}{x+7}$$

$$= (x+7)(x-7)\frac{(5x+2)}{(x+7)}$$

$$= (x-7)(5x+2) \textbf{ Ans}$$

(ii) $\dfrac{4x-12}{x^2-9} \div \dfrac{18-2x^2}{x^2+6x+9}$

Solution: $\dfrac{4x-12}{x^2-9} \div \dfrac{18-2x^2}{x^2+2(x)(3)+(3)^2}$

$$= \frac{4(x-3)}{(x^2)-(3)^2} \div \frac{2(9-x^2)}{(x+3)^2}$$

$$= \frac{4\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} \times \frac{(x+3)^2}{2(9-x^2)}$$

$$= \frac{4}{x+3} \times \frac{(x+3)^2}{2(3+x)(3-x)}$$

$$= \frac{^{2}\cancel{4}x\cancel{(x+3)^2}}{\cancel{2}\cancel{(x+3)^2}(3-x)}$$

$$= \frac{2}{3-x} \textbf{ Ans}$$

(iii) $\dfrac{x^6-y^6}{x^2-y^2} \div \left(x^4+x^2y^2+y^4\right)$

Solution: $\dfrac{x^6-y^6}{x^2-y^2} \div \left(x^4+x^2y^2+y^4\right)$

$$= \frac{(x^2)^3-(y^2)^3}{x^2-y^2} \div \left(x^4+x^2y^2+y^4\right)$$

$$= \frac{\cancel{(x^2-y^2)}\left[(x^2)^2+x^2y^2+(y^2)^2\right]}{\cancel{(x^2-y^2)}} \div \left(x^4+x^2y^2+y^4\right)$$

$$= \cancel{\left(x^4+x^2y^2+y^4\right)} \times \frac{1}{\cancel{\left(x^4+x^2y^2+y^4\right)}}$$

$= 1$ **Ans**

(iv) $\dfrac{x^2-1}{x^2+2x+1}.\dfrac{x+5}{1-x}$

Solution: $\dfrac{x^2-1}{x^2+2x+1}.\dfrac{x+5}{1-x}$

$$= \frac{(x+1)(x-1)}{\left(x^2+2(x)(1)+(1)^2\right)} \times \frac{x+5}{-(x-1)}$$

$$= \frac{(x+1)\cancel{(x-1)}}{(x+1)^2} \times \frac{(x+5)}{\cancel{-(x-1)}}$$

$$= -\frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x+1)}$$

$$= -\frac{(x+5)}{x+1}$$ **Ans**

(v) $$\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$$

Solution: $$\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$$

$$= \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \cdot \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \div \frac{x(x-1)}{y(x-2)}$$

$$= \frac{x.\cancel{x}}{y.\cancel{y}} \times \frac{\cancel{y}(x-2)}{\cancel{x}(x-1)}$$

$$= \frac{x(x-2)}{y(x-1)}$$ **Ans**

Exercise 4.2

Q.1 Solve

(i) If $a+b=10$ and $a-b=6$, then find the value of $\left(a^2+b^2\right)$

Solution:

$$2\left(a^2+b^2\right)=(a+b)^2+(a-b)^2$$
$$2\left(a^2+b^2\right)=(10)^2+(6)^2$$
$$2\left(a^2+b^2\right)=100+36$$
$$2\left(a^2+b^2\right)=136$$
$$\left(a^2+b^2\right)=\frac{\cancel{136}^{68}}{\cancel{2}}$$
$$\left(a^2+b^2\right)=68 \text{ Ans}$$

(ii) If $a+b=5, a-b=\sqrt{17}$, then find the value of ab.

Solution:

$$4ab=(a+b)^2-(a-b)^2$$
$$4ab=(5)^2-\left(\sqrt{17}\right)^2$$
$$4ab=25-17$$
$$4ab=8$$
$$ab=\frac{8}{4}$$
$$ab=2$$
$$ab=2 \text{ Ans}$$

Q.2 If $a^2+b^2+c^2=45$ and $a+b+c=-1$, then find the value of $ab+bc+ca$.

Solution: $a^2+b^2+c^2=45$

$a+b+c=-1$

$ab+bc+ca$ **=?**

We know that

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$
$$(-1)^2=45+2(ab+bc+ca)$$
$$1=45+2(ab+bc+ca)$$
$$1-45=2(ab+bc+ca)$$
$$-44=2(ab+bc+ca)$$
$$\frac{\cancel{-44}^{-22}}{\cancel{2}}=(ab+bc+ca)$$
$$(ab+bc+ca)=-22 \text{ Ans}$$

Q.3 If $m+n+p=10$ and $mn+np+np=27$, find the value of $m^2+n^2+p^2$

Solution: $m+n+p=10$

$mn+np+np=27,$

$m^2+n^2+p^2$ **=?**

We know that

$$(m+n+p)^2=m^2+n^2+p^2+2mn+2np+2mp$$
$$(10)^2=m^2+n^2+p^2+2(mn+np+mp)$$
$$100=m^2+n^2+p^2+2(27)$$
$$100=m^2+n^2+p^2+54$$
$$100-54=m^2+n^2+p^2$$
$$m^2+n^2+p^2=46 \text{ Ans}$$

Q.4 If $x^2+y^2+z^2=78$ and $xy+yz+zx=59$, find the value of $x+y+z$.

Solution: $x^2+y^2+z^2=78$

$xy+yz+zx=59,$

$x+y+z$ **=?**

We know that

$$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$$
$$(x+y+z)^2=78+2(xy+yz+zx)$$
$$(x+y+z)^2=78+2(59)$$
$$(x+y+z)^2=78+118$$
$$(x+y+z)^2=196$$

Taking square root at both sides